MÉTODOS NUMÉRICOS

Ec. calor: $\rho c \cdot u_t - \tau \cdot u_{xx} = f(x,t)$

Sol. de una ec. diferencial lineal homogénea de 2° orden con coef. constantes.

$y = y(x)$ $\qquad y'' + a_1 \cdot y' + a_0 = 0$

$$\lambda^2 + a_1 \cdot \lambda + a_0 = 0 \begin{cases} \lambda_1 \\ \lambda_2 \end{cases}$$

<u>1^er caso</u> $\lambda_1 = \lambda_2$ (real)

$$y = c_1 \cdot e^{x \cdot \lambda_1} + c_2 x e^{x \cdot \lambda_1}$$

<u>2° caso</u> $\lambda_1 \neq \lambda_2$ (real)

$$y = c_1 \cdot e^{\lambda_1 \cdot x} + c_2 \cdot e^{\lambda_2 \cdot x}$$

<u>3^er caso</u> $\lambda_1 = a + bi$
$\lambda_2 = a - bi$

$$y = c_1 \cdot e^{ax} \cdot \cos bx + c_2 \cdot e^{ax} \cdot \operatorname{sen} bx$$

Serie de Fourier:

$(-L, L)$ Intervalo.
$\parallel$
Int

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cdot \cos nx + b_n \cdot \operatorname{sen} nx \right]$$

$$a_n = \frac{1}{Int/2} \int_{-L}^{L} f(x) \cdot \cos nx \cdot dx$$

$$b_n = \frac{1}{Int/2} \int_{-L}^{L} f(x) \cdot \operatorname{sen} nx \cdot dx$$

$$\frac{a_0}{2} = \frac{1}{Int} \int_{-L}^{L} f(x) \cdot dx$$

condición térmica

$\rho c \cdot u_t - \kappa u_{xx} = f(x,t)$ $0 < x < 1$; $t > 0$

ρ = densidad

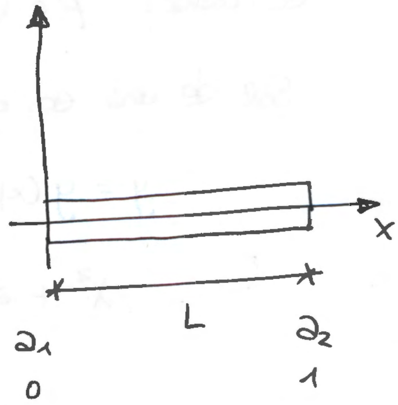
c = calor específico

κ = cond. térmica

u = función t^a

$u(x,t)$

posición   temperatura



$f(x,t)$: > 0 : fuente calor

< 0 : sumidero calor

C.C. 1

C.C. 2

condición contorno tipo DIRICHLET.

$u(a_1,t) = g_1(t)$

$u(a_2,t) = g_2(t)$

C.C. Tipo Newman

$\kappa u_x(a,t) = g_1(t)$

condición inicial:

$u(x,0) = h_0(x)$

1 EC. CALOR. SOLUCIÓN EXACTA.

$\rho c \cdot u_z - \tau u_{xx} = f(x,t)$

$u_t - u_{xx} = 0 \quad 0<x<1 \;;\; t>0$

$\left.\begin{array}{l} u(0,t)=0 \\ u(1,t)=0 \end{array}\right\} t>0$

$u(x,0) = \operatorname{sen} \pi x \quad 0<x<1$

a) Interpretación física.

b) Solución exacta. $p\nexists\ f(x,t)=0$

No hay fuentes ni sumideros de calor en el interior de la barra. Inicialmente, la temperatura de los puntos interiores de la barra valen $\operatorname{sen}\pi x$.

M.S.V (método separación de variables).

$\boxed{u(x,t) = X(x)\cdot T(t)}$ ← solución que buscamos.

$$\left.\begin{array}{l} u_t : \underbrace{\cancel{X_t}}_{0}\, T + X\cdot T' \\ u_x = X'\cdot T + X\cdot \underbrace{\cancel{T_x}}_{0} \\ u_{xx} = X''\cdot T + X'\cdot \underbrace{\cancel{T_x}}_{0} \end{array}\right\}$$

$X\cdot T' - X''\cdot T = 0$

$\dfrac{X''}{X} = \dfrac{T'}{T} = k$ ← constante de separación.

$$\begin{cases} X'' - kX = 0 \\ \dfrac{T'}{T} = k \rightarrow T = C_3\cdot e^{kt} \end{cases}$$

1er caso $k=0$.

$X''=0 \rightarrow \lambda^2=0 \rightarrow$ solución real y única

$X = C_1\cdot e^{0x} + C_2\cdot x\cdot e^{0x} = C_1 + C_2\cdot x$

$$\begin{cases} u(0,t)=0 = X(0)\cdot T(t) \quad \boxed{X(0)=0} \quad T(t)=0 \rightarrow \text{soluc trivial.} \\ u(1,t)=0 = X(1)\cdot T(t) \quad X(1)=0 \quad T(t)=0 \rightarrow \text{sol trivial.} \end{cases}$$

$$\left.\begin{array}{l} X(0)=0 = C_1 + C_2\, 0 \rightarrow C_1=0 \\ X(1)=0 = C_2\cdot 1 \rightarrow C_2=0 \end{array}\right\} X=0$$

$u(x,t) = \underset{0}{X(x)}\cdot \underset{0}{T(t)} = 0$

2º caso $(k>0)$

$$X'' - kX = 0$$

$$\lambda^2 - k = 0 \rightarrow \lambda = \pm\sqrt{k}$$

$$X = C_1 \cdot e^{\sqrt{k}\cdot x} + C_2 \cdot e^{-\sqrt{k}\cdot x}$$

$$\left.\begin{array}{l} X(0) = 0 = C_1 \cdot e^0 + C_2 \cdot e^0 \\ X(1) = 0 = C_1 \cdot e^{\sqrt{k}} + C_2 \cdot e^{-\sqrt{k}} \end{array}\right\} \quad \begin{array}{l} C_1 = C_2 = 0 \\ u(x,t) = 0 \end{array}$$

3er caso $(k<0)$

$$X'' - kX = 0 \rightarrow \lambda^2 - k = 0 \rightarrow \lambda = \pm\sqrt{k} = \pm\sqrt{-k}\cdot i$$

$$X = C_1 \cdot e^{0x} \cdot \cos\sqrt{-k}\cdot x + C_2 \cdot e^{0x} \cdot \operatorname{sen}\sqrt{-k}\cdot x$$

$$X(0) = 0 = C_1 \cdot \underset{1}{\cos 0} + C_2 \cdot \underset{0}{\operatorname{sen} 0} \rightarrow C_1 = 0$$

$X(1) = 0 = C_2 \cdot \operatorname{sen}\sqrt{-k}\cdot 1$ $\begin{cases} C_2 = 0 \rightarrow \text{sol. trivial.} \\ \operatorname{sen}\sqrt{-k}\cdot 1 = 0 \end{cases}$

$$\operatorname{sen}\sqrt{-k}\cdot 1 = 0$$

$$\sqrt{-k}\cdot 1 = \frac{n\pi}{1} \;;\; n = 1,2,3\ldots \qquad k = -n^2\pi^2$$

$$X = C_2 \cdot \operatorname{sen} n\pi x \longrightarrow X = \sum_{n=1}^{\infty} C_2 \operatorname{sen} n\pi x$$

$$T = C_3 \cdot e^{-(n\pi)^2 t} \longrightarrow T = \sum_{n=1}^{\infty} C_3 \cdot e^{-(n\pi)^2 t}$$

$$u(x,t) = \sum_{n=1}^{\infty} D_n \cdot \operatorname{sen} n\pi x \cdot e^{-(n\pi)^2 t}$$

$$u(x,0) = \operatorname{sen}\pi x = \sum_{n=1}^{\infty} D_n \cdot \operatorname{sen} n\pi x \cdot e^0$$

$$\operatorname{sen}\pi x = \sum_{n=1}^{\infty} D_n \cdot \operatorname{sen} n\pi x \longrightarrow \text{Serie de Fourier.}$$

$$\operatorname{sen}\pi x = D_1 \cdot \operatorname{sen}\pi x + D_2 \cdot \operatorname{sen} 2\pi x + D_3 \cdot \operatorname{sen} 3\pi x + \cdots$$

$$D_1 = 1$$

$$D_n\,(n \neq 1) = 0$$

$$D_n = \frac{1}{1/2}\int_0^1 sen\,\pi x \cdot sen\, n\pi x \cdot dx$$

$$D_n = \cancel{2}\int_0^1 \frac{1}{\cancel{2}}\left[\cos(1-n)\pi x - \cos(1+n)\pi x\right]\cdot dx =$$

$$= \frac{1}{(1-n)\pi}\int_0^1 (1-n)\pi \cdot \cos(1-n)\pi x \cdot dx - \frac{1}{(1+n)\pi}\int_0^1 (1+n)\pi \cdot \cos(1+n)\pi x \cdot dx =$$

$$= \frac{1}{(1-n)\pi}\cdot sen(1-n)\pi x\Big]_0^1 - \frac{1}{(1+n)\pi}\cdot sen(1+n)\pi x\Big]_0^1 =$$

$$= \frac{1}{(1-n)\pi}\Big[sen(1-n)\pi - \underbrace{\cancel{sen}\,0}_{0}\Big] - \frac{1}{(1+n)\pi}\Big[sen(1+n)\pi - \underbrace{\cancel{sen}\,0}_{0}\Big]$$

Acaba aquí el problema

② $\rho c \cdot u_t - \tau u_{xx} = f(x,t)$

$u_t - u_{xx} = 0 \qquad 0 < x < 1 \; ; \; t > 0$

$\left.\begin{array}{l} u_x(0,t) = 0 \\ u_x(1,t) = 0 \end{array}\right\} t > 0$

$u(x,0) = \operatorname{sen} \pi x \qquad 0 < x < 1$

a) Interpretación física.

b) Solución exacta.

No tengo fuentes ni sumideros de calor. (pq $f(x,t) = 0$)
Inicialmente, la temperatura de los puntos interiores de la barra valen sen πx.

MSV

$u(x,t) = X(x) \cdot T(t)$

$X \cdot T' - X'' \cdot T = 0$

$\frac{X''}{X} = \frac{T'}{T} = k$

$X'' - kX = 0$

$\frac{T'}{T} = k$

Derivando x respecto de t

$u_x(x,t) = X'(x) \cdot T(t) + X(x) \cdot \cancel{T_x}(t)$ → 0

$u_x(0,t) = 0 = X'(0) \cdot T(t)$ — $X'(0) = 0$; $T(t) = 0 \to$ S.T.

$u_x(1,t) = 0 = X'(1) \cdot T(t)$ — $X'(1) = 0$; $T(t) = 0 \to$ S.T.

1er caso $(k = 0)$

$X'' = 0$

$\lambda^2 = 0 \to \lambda = \pm 0$

$X = C_1 \cdot e^{0x} + C_2 \cdot x \cdot e^{0x} = C_1 + C_2 \cdot x$

$\left.\begin{array}{l} X' = C_2 \\ X'(0) = 0 = C_2 \\ X'(1) = 0 = C_2 \end{array}\right\} X = C_1$

$\frac{T'}{T} = k \to T = C_3 \cdot e^{kt} = C_3 \cdot e^0 = C_3$

$\boxed{u(x,t) = C_1 \cdot C_3 = A}$

2° caso ($k>0$)

$$X'' - kX = 0$$

$$\lambda^2 - k = 0;\ \lambda = \pm\sqrt{k}$$

$$X = C_1 \cdot e^{\sqrt{k}\cdot x} + C_2 \cdot e^{-\sqrt{k}\cdot x}$$

$$X' = C_1 \cdot \sqrt{k} \cdot e^{\sqrt{k}\cdot x} - C_2 \cdot \sqrt{k} \cdot e^{-\sqrt{k}\cdot x}$$

$$\left.\begin{array}{l} X'(0) = 0 = C_1 \cdot \sqrt{k} \cdot e^{0} - C_2 \cdot \sqrt{k} \cdot e^{0} \\ X'(1) = 0 = C_1 \cdot \sqrt{k} \cdot e^{\sqrt{k}} - C_2 \cdot \sqrt{k} \cdot e^{-\sqrt{k}} \end{array}\right\} \begin{array}{l} C_1 = C_2 = 0 \\ \boxed{X = 0} \\ \boxed{u(x,t) = 0} \end{array}$$

3er caso ($k<0$)

$$X'' - kX = 0$$

$$\lambda^2 - k = 0;\ \lambda = \pm\sqrt{k} = \pm\sqrt{-k}\cdot i$$

$$X = C_1 \, e^{0x} \cdot \cos\sqrt{-k}\cdot x + C_2 \cdot e^{0x} \cdot \operatorname{sen}\sqrt{-k}\cdot x$$

$$X' = -C_1 \sqrt{-k} \cdot \operatorname{sen}\sqrt{-k}\cdot x + C_2 \cdot \sqrt{-k} \cdot \cos\sqrt{-k}\cdot x$$

$$X'(0) = 0 = -C_1 \cdot \sqrt{-k} \cdot \operatorname{sen} 0 + C_2 \cdot \sqrt{-k} \cdot \cos 0 \begin{cases} C_2 = 0 \\ \sqrt{-k} = 0 \to k = 0 \ \underline{\text{No}}\ !! \end{cases}$$

$$X'(1) = 0 = -C_1 \cdot \sqrt{-k} \cdot \operatorname{sen}\sqrt{-k}\cdot 1 \begin{cases} C_1 = 0 \to \boxed{X = 0} \to \text{S.T.} \\ \sqrt{-k} = 0 \to \underline{\text{No}}\ !! \\ \operatorname{sen}\sqrt{-k} = 0 \to \sqrt{-k} = n\pi \ ;\ n = 1, 2 \ldots \end{cases}$$

$$\downarrow$$

$$k = -(n\pi)^2$$

$$X = C_1 \cdot \cos n\pi x \longrightarrow X = \sum_{n=1}^{\infty} C_1 \cdot \cos n\pi x$$

$$T = C_3 \cdot e^{-(n\pi)^2 t} \longrightarrow T = \sum_{n=1}^{\infty} C_3 \cdot e^{-(n\pi)^2 t}$$

$$\boxed{u(x,t) = \sum_{n=1}^{\infty} D_n \cdot \cos n\pi x \cdot e^{-(n\pi)^2 t}}$$

$$u(x,t) = \underset{\substack{\uparrow \\ 1^{er}\text{caso}}}{A} + \underset{\substack{\uparrow \\ 2^{o}\text{caso}}}{0} + \sum_{n=1}^{\infty} D_n \cdot \cos n\pi x \cdot e^{-(n\pi)^2 t}$$

Aplicamos condiciones iniciales:

$$u(x,0) = \operatorname{sen} \pi x = A + \sum_{n=1}^{\infty} D_n \cdot \cos n\pi x \cdot e^{0}$$

$$A = \frac{1}{1}\int_0^1 \operatorname{sen}\pi x \cdot dx = \frac{1}{\pi}\int_0^1 \pi \cdot \operatorname{sen}\pi x \cdot dx = -\frac{1}{\pi}\cdot \cos\pi x\Big|_0^1 = -\frac{1}{\pi}\Big[\underset{-1}{\cos\pi} - \underset{1}{\cos 0}\Big] = \frac{2}{\pi}$$

$$D_n = \frac{1}{1/2}\int_0^1 \operatorname{sen}\pi x \cdot \cos n\pi x \cdot dx = 2\int_0^1 \frac{1}{2}\left[\operatorname{sen}(1-n)\pi x + \operatorname{sen}(1+n)\pi x\right]\cdot dx =$$

$$= \frac{1}{(1-n)\pi}\int_0^1 (1-n)\pi \cdot \operatorname{sen}(1-n)\pi x \cdot dx + \frac{1}{(1+n)\pi}\int_0^1 (1+n)\pi \cdot \operatorname{sen}(1+n)\pi x \cdot dx =$$

$$= \frac{-1}{(1-n)\pi}\cdot \cos(1-n)\pi x\Big|_0^1 - \frac{1}{(1+n)\pi}\cdot\cos(1+n)\pi x\Big|_0^1 =$$

$$= \underbrace{\frac{-1}{(1-n)\pi}\left[\cos(1-n)\pi - \cos 0\right] - \frac{1}{(1+n)\pi}\left[\cos(1+n)\pi - \cos 0\right]}$$

$$\left[u(x,t) = \underset{A}{\frac{2}{\pi}} + \sum_{n=1}^{\infty}\left[D_n\right]\cdot \cos n\pi x \cdot e^{-(n\pi)^2 t}\right] \quad \text{solución}$$

$\rho c \cdot u_t - \tau u_{xx} = f(x,t)$

③

① $u_t - 7u_{xx} = 0 \qquad 0<x<\pi\,;\ t>0$

② $\left.\begin{cases} u(0,t)=0 \\ u(\pi,t)=0 \end{cases}\right\} t>0$

③ $u(x,0) = 3\cdot \text{sen}\, 2x - 6\cdot \text{sen}\, 5x \qquad 0<x<\pi$

a) Interpretación física.

b) Solución exacta.

No hay fuentes ni sumideros de calor. (pq $f(x,t)=0$)
Inicialmente, la temperatura vale $3\,\text{sen}\,2x - 6\cdot\text{sen}\,5x$.

MSV

$u(x,t) = X(x)\cdot T(t)$

$X\cdot T' - 7X''\cdot T = 0$

$\dfrac{X''}{X} = \dfrac{T'}{7T} = k \rightarrow X'' - kX = 0$

$\dfrac{T'}{T} = 7k \rightarrow T = C_3 \cdot e^{7kt}$

$u(0,t) = 0 = X(0)\cdot T(t) \begin{cases} X(0)=0 \\ T(t)=0 \rightarrow \text{S.T.} \end{cases}$

$u(\pi,t) = 0 = X(\pi)\cdot T(t) \begin{cases} X(\pi)=0 \\ T(t)=0 \rightarrow \text{S.T.} \end{cases}$

1er caso $(k=0)$

$X''=0$

$\lambda^2 = 0\,;\ \lambda = \pm 0$

$X = C_1\cdot e^{0x} + C_2\cdot x\cdot e^{0x} = C_1 + C_2\cdot x$

$\left.\begin{matrix} X(0) = 0 = C_1 + 0 \rightarrow C_1 = 0 \\ X(\pi) = 0 = C_2\cdot\pi \rightarrow C_2 = 0 \end{matrix}\right\} \begin{matrix} X = 0 \\ u(x,t) = 0 \end{matrix}$

2º caso $(k>0)$

$$X'' - kX = 0$$

$$\lambda^2 - k = 0 \rightarrow \lambda = \pm\sqrt{k}$$

$$X = C_1 \cdot e^{\sqrt{k}\, x} + C_2 \cdot e^{-\sqrt{k} \cdot x}$$

$$\left.\begin{array}{l} X(0) = 0 = C_1 \cdot e^0 + C_2 \cdot e^0 \\ X(\pi) = 0 = C_1 \cdot e^{\sqrt{k} \cdot \pi} + C_2 \cdot e^{-\sqrt{k} \cdot \pi} \end{array}\right\} \begin{array}{l} C_1 = C_2 = 0 \\ X = 0 \\ u(x,t) = 0 \end{array}$$

3er caso $(k<0)$

$$X'' - kX = 0$$

$$\lambda^2 - k = 0 \rightarrow \lambda = \pm\sqrt{k} = \pm\sqrt{-k} \cdot i$$

$$X = C_1 \cdot e^{0x} \cdot \cos\sqrt{-k} \cdot x + C_2 \cdot e^{0x} \cdot \operatorname{sen}\sqrt{-k} \cdot x$$

$$X(0) = 0 = C_1 \cdot \underset{1}{\underbrace{\cos 0}} + C_2 \cdot \underset{0}{\underbrace{\operatorname{sen} 0}} \rightarrow C_1 = 0$$

$$X(\pi) = 0 = C_2 \cdot \operatorname{sen}\sqrt{-k} \cdot \pi \quad \begin{array}{l} C_2 = 0 \\ \operatorname{sen}\sqrt{-k} \cdot \pi = 0 \end{array}$$

$$\sqrt{-k} \cdot \cancel{\pi} = n\cancel{\pi} \quad ; \quad n = 1, 2 \ldots$$

$$\hookrightarrow k = -n^2$$

$$X = C_2 \cdot \operatorname{sen} nx \rightarrow X = \sum_{n=1}^{\infty} C_2 \cdot \operatorname{sen} nx$$

$$T = C_3 \cdot e^{-7n^2 t} \longrightarrow T = \sum_{n=1}^{\infty} C_3 \cdot e^{-7n^2 t}$$

$$u(x,t) = \sum_{n=1}^{\infty} D_n \cdot \text{sen}\, nx \cdot e^{-7n^2 t}$$

$$u(x,0) = 3\,\text{sen}\, 2x - 6\cdot\text{sen}\, 5x = \sum_{n=1}^{\infty} D_n \cdot \text{sen}\, nx \cdot \cancel{e^{0}}\, 1$$

$$3\,\text{sen}\, 2x - 6\,\text{sen}\, 5x = D_1 \cdot \text{sen}\, x + D_2 \cdot \text{sen}\, 2x + D_3 \cdot \text{sen}\, 3x + D_4 \cdot \text{sen}\, 4x + D_5 \cdot \text{sen}\, 5x + \ldots$$

$D_2 = 3$

$D_5 = 6$

$D_n\ (n \neq 2,5) = 0$

$$\left[\, u(x,t) = \underset{n=1}{0} + \underset{n=2}{3\,\text{sen}\, 2x \cdot e^{-7\cdot 4t}} + \underset{n=3}{0} + \underset{n=4}{0} - \underset{n=5}{6\cdot\text{sen}\, 5x \cdot e^{-7\cdot 25t}} + \ldots \,\right]$$

solución

Homogeneización : Resolver por el método de separación de variables (MSV)

13.09.2016

8.

$P(u) = f(x,t)$

$$\boxed{PC \cdot u_t - 7u_{xx}} = f(x,t)$$

$CC \cdot 1 = g_1(t)$

$CC2 = g_2(t)$

$CI = h_1(x)$

cond. iniciales

Problema no homogéneo (PNH) $\rightarrow u$

u_p es solución PNH

$P(u_h) = 0$

$$P_C \cdot u_t - 7u_{xx} = 0 \quad (PH)$$

$CC1 = 0$

$CC2 = 0$

$CI = h_0(x)$

problema homogéneo (MSV) $\rightarrow u_h$.

$u = u_h + u_p$

Si: $\left.\begin{array}{l} u(a,t) = g_1(t) \\ u(b,t) = g_2(t) \end{array}\right\}$ $u_p = \dfrac{b-x}{b-a} \cdot g_1(t) + \dfrac{x-a}{b-a} \cdot g_2(t)$

tipo Newman (las 2 con derivada).

Si: $\left.\begin{array}{l} u_x(a,t) = q_1(t) \\ u_x(b,t) = q_2(t) \end{array}\right\}$ $u_p = \dfrac{-(b-x)^2}{2(b-a)} \cdot q_1(t) + \dfrac{(x-a)^2}{2(b-a)} \cdot q_2(t)$

tipo Dirichlet.

Si: $\left.\begin{array}{l} u(a,t) = g(t) \\ u_x(b,t) = q(t) \end{array}\right\}$ $u_p = (x-a)\,q(t) + g(t)$

Ejemplo:

$\left.\begin{array}{l} u(0,t) = t^2 \\ u_x(\pi,t) = 1+t \end{array}\right\}$ $u_p = (x-\pi)(1+t)+t^2$

$u = u_h + u_p$

$u_h = u - u_p$

$$\boxed{P(u_h) = P(u-u_p) = P(u) - P(u_p)} = 0$$

4 a)

$P \to u_t - u_{xx} = 2t(\pi - x) \qquad 0<x<\pi \ ; \ t>0$

$Q \to u(0,t) = \pi t^2$
$R \to u(\pi,t) = 0$ $\Big\} \ t>0$

$S \to u(x,0) = \begin{cases} x & 0< x \leq \pi/2 \\ \pi - x & \pi/2 < x < \pi \end{cases}$

$u_p = (\pi - x)\, t^2$

$u_p = \dfrac{\pi - x}{\pi - 0} \cdot \pi t^2 + \dfrac{x - 0}{\pi - 0} \cdot 0 = (\pi - x)\cdot t^2$

$u_h = u - u_p$

$\boxed{P(u_h)} = P(u) - P(u_p) = 2t(\pi - x) - (\pi - x)\cdot 2t = 0$

$u_{ht} - u_{hxx} = 0$

$P(u_p) = u_{p_t} - u_{p_{xx}} = \underbrace{(\pi - x)\cdot 2t}_{u_{pt}} - \underset{u_{pxx}}{0} \to u_{ht} - u_{hxx} = 0$

$Q(u_p) = (\pi - 0)t^2$

$R(u_p) = (\pi - \pi)t^2 = 0$

$\boxed{Q(u_h)} = Q(u) - Q(u_p) = \pi t^2 - \pi t^2 = 0 \to u_h(0,t) = 0$

$\boxed{R(u_h)} = R(h) - R(u_p) = 0 - 0 = 0 \to u_h(\pi,t) = 0$

$S(u_h) = S(u) - S(u_p) = \begin{cases} x - 0 = x \\ \pi - x - 0 = \pi - x \end{cases}$

$u_h(x,0) = \begin{cases} x \\ \pi - x \end{cases}$

M.S.V.

$u = u_{ht}\ (\pi - x)t^2$

CASUALIDAD

b) P: $u_t - u_{xx} = 4t^3(x-\pi)$ $\quad 0<x<\pi\ ;\ t>0$

Q: $u_x(0,t) = 1+t^4$
R: $u(\pi,t) = 0$ $\quad \Big\} \ t>0$

S: $u(x,0) = x-\pi$ $\quad 0<x<\pi$

$P(u_p) = u_{p_t} - u_{p_{xx}} = (x-\pi)\,4t^3 \neq 0$

$Q(u_p) = 1+t^4$

$R(u_p) = 0$

$u_p = (x-\pi)(1+t^4) + 0$

$u_{p_x} = 1+t^4$

$P(u_h) = P(u) - P(u_p) = 4t^3(x-\pi) - (x-\pi)4t^3 = 0 \rightarrow u_{h_t} - u_{h_{xx}} = 0$

$Q(u_h) = Q(u) - Q(u_p) = 1+t^4 - (1+t^4) = 0 \rightarrow u_{h_x}(0,t) = 0$

$R(u_h) = R(u) - R(u_p) = 0 - 0 = 0 \rightarrow u_h(\pi,t) = 0$

$S(u_h) = S(u) - S(u_p) = x-\pi-(x-\pi) = 0 \rightarrow u_h(x,0) = 0$

$\rightarrow$ MSV $\quad u_h = 0$

$$\boxed{u = \underset{u_h}{0} + \underbrace{(x-\pi)(1+t^4)}_{u_p}}$$ $\leftarrow$ solución única

16.09.2016

5 $P \to u_t - u_{xx} = 2xt \qquad 0 < x < 1 \; ; \; t > 0$

$$\left.\begin{array}{l} Q \to u(0,t) = 1 \\ R \to u(1,t) = t^2 \end{array}\right\} t > 0$$

$S \to u(x,0) = \cos \pi x + 1 - x \qquad 0 < x < 1$

a) Interpretación física.

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- para $t = 0$; la temp. de los puntos del interior de la barra vale $[\cos \pi x + 1 - x]$

b) Tenemos que homogeneizar, aunque no lo diga el enunciado.

$$u_p = \frac{b - x}{b - a} g_1(t) + \frac{x - a}{b - a} \cdot g_2(t)$$

$$u_p = \frac{1 - x}{1 - 0} \cdot 1 + \frac{x - 0}{1 - 0} \cdot t^2 = 1 - x + xt^2$$

$$P(u_h) = P(u) - P(u_p) = 2xt - 2xt = 0 \to \boxed{\begin{array}{l} u_{ht} - u_{hxx} = 0 \\ 0 < x < 1 \; ; \; t > 0 \end{array}}$$

$$P(u_p) = u_{pt} - u_{pxx} = 2xt - 0$$

$$Q(u_h) = Q(u) - Q(u_p) = 1 - 1 = 0 \to \boxed{u_h(0,t) = 0} * \; t > 0$$

$$Q(u_p) = 1$$

$R(u_h) = R(u) - R(u_p) = t^2 - t^2 = 0 \rightarrow \boxed{u_h(1,t) = 0}$ * $t>0$

* $\} \; t>0$

$R(u_p) = t^2$

$S(u_h) = S(u) - S(u_p) = \cos\pi x + 1 - x - (1-x) = \cos\pi x$

$S(u_p) = 1-x$

$\boxed{u_h(x,0) = \cos\pi x \; ; \; 0<x<1}$

M.S.V.

$u_h(x,t) = X(x)\cdot T(t)$

$X\cdot T' - X''\cdot T = 0$

$$\frac{X''}{X} = \frac{T'}{T} = k$$

$X'' - kX = 0$

$\frac{T'}{T} = k \; ; \; T = C_3\cdot e^{kt}$

$u_h(0,t) = 0 = X(0)\cdot T(t)$

se aplica el M.S.V. a las que están marcadas (pq tienen que estar igualadas a cero).

$X(0) = 0$

$T(t) = 0$ S.T.

$u_h(1,t) = 0 = X(1)\cdot T(t)$ $\begin{cases} X(1) = 0 \\ T(t) = 0 \quad \text{S.T.} \end{cases}$

1er caso $k=0$

$X'' = 0$

$\lambda^2 = 0 \rightarrow \lambda = \pm 0$

$X = C_1\cdot e^{0x} + C_2\cdot x\cdot e^{0x} = C_1 + C_2\cdot x$

~~$X(0) = 0 = C_1\cdot e^{0x} + C_2\cdot x\cdot e^{0x} = C_1 + C_2 x$~~

$X(0) = 0 = C_1 + C_2\cdot 0 \rightarrow C_1 = 0$ $\qquad X = 0$

$X(1) = 0 = C_2\cdot 1 \rightarrow C_2 = 0$ $\qquad u_h(x,t) = 0$

2º caso $k>0$

$$X'' - kX = 0$$

$$\lambda^2 - k = 0; \quad \lambda = \pm\sqrt{k}$$

$$X = C_1 \cdot e^{\sqrt{k}\cdot x} + C_2 \cdot e^{-\sqrt{k}\cdot x}$$

$$\left.\begin{array}{l} X(0) = 0 = C_1 \cdot e^0 + C_2 \cdot e^0 \\ X(1) = 0 = C_1 \cdot e^{\sqrt{k}} + C_2 \cdot e^{-\sqrt{k}} \end{array}\right\} \quad \begin{array}{l} C_1 = C_2 = 0 \\ X = 0 \\ u_k(x,t) = 0 \end{array}$$

3er caso $k<0$

$$X'' - kX = 0$$

$$\lambda^2 - k = 0; \quad \lambda = \pm\sqrt{k} = \pm\sqrt{-k}\, i$$

$$X = C_1 \cdot e^{0x} \cdot \cos\sqrt{-k}\cdot x + C_2 \cdot e^{0x} \cdot \operatorname{sen}\sqrt{-k}\cdot x$$

$$\left.\begin{array}{l} X(0) = 0 = C_1 \cdot \underset{1}{\cos 0} + C_2 \cdot \underset{0}{\operatorname{sen} 0} \to C_1 = 0 \\ X(1) = 0 = C_2 \cdot \operatorname{sen}\sqrt{-k} \left\langle \begin{array}{l} C_2 = 0 \\ \operatorname{sen}\sqrt{-k} = 0 \end{array}\right. \end{array}\right\} \quad \begin{array}{l} X = 0 \\ \text{S.T.} \end{array}$$

$$\operatorname{sen}\sqrt{-k} = 0$$

$$\sqrt{-k} = n\pi; \quad n = 1, 2 \ldots$$

$$X = C_2 \cdot \operatorname{sen} n\pi x$$

$$X = \sum_{n=1}^{\infty} C_2 \cdot \operatorname{sen} n\pi x$$

$$T = C_3 \cdot e^{-(n\pi)^2 t}$$

$$T = \sum_{n=1}^{\infty} C_3 \cdot e^{-(n\pi)^2 t}$$

$$u_h(x,t) = \sum_{n=1}^{\infty} D_n \cdot \text{sen}\, n\pi x \cdot e^{-(n\pi)^2 t}$$

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$$u_h(x,0) = \boxed{\cos \pi x} = \sum_{n=1}^{\infty} D_n \cdot \text{sen}\, n\pi x \cdot e^{0}$$ Serie de Fourier.

$$D_n = \frac{1}{1/2}\int_0^1 \cos \pi x \cdot \text{sen}\, n\pi x \cdot dx = 2 \cdot \int_0^1 \frac{1}{2}\Big[\text{sen}(1-n)\pi x +$$

$$+ \text{sen}\,(1+n)\pi x\Big] \cdot dx =$$

$$= \frac{1}{(1-n)\pi}\int_0^1 (1-n)\pi \cdot \text{sen}(1-n)\pi x \cdot dx + \frac{1}{(1+n)\pi}\int_0^1 (1+n)\pi \cdot \text{sen}\,(1+n)\pi x\, dx =$$

$$= \frac{-1}{(1-n)\pi} \cdot \cos(1-n)\pi x\Big|_0^1 - \frac{1}{(1+n)\pi} \cdot \cos(1+n)\pi x\Big|_0^1 =$$

$$= \boxed{\frac{-1}{(1-n)\pi}\Big[\cos(1-n)\pi - \overset{1}{\cos 0}\Big] - \frac{1}{(1+n)\pi}\Big[\cos(1+n)\pi - \cos 0\Big]}$$

$$\| \quad D_n$$

$$u_h = \sum_{n=1}^{\infty} D_n \cdot \text{sen}\, n\pi x \cdot e^{-(n\pi)^2 t}$$

$$u = 1 - x + xt^2 + \sum_{n=1}^{\infty} D_n \cdot \text{sen}\, n\pi x \cdot e^{-(n\pi)^2 t}$$

EC. CALOR. SOLUCIÓN APROXIMADA

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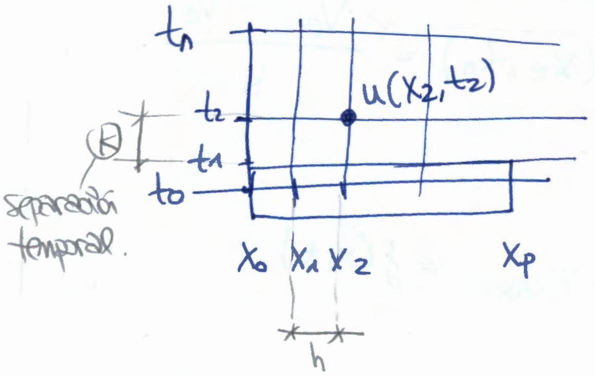
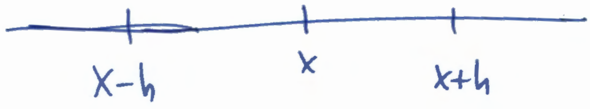


$$\rho c\, u_t - \tau u_{xx} = f(x,t)$$

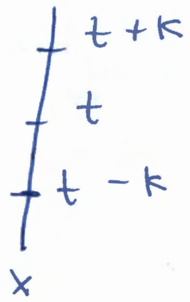
CC.1

CC.2

CC.3



$\cancel{u(x+h,t)} =$



$$u(x+h,t) = u(x,t) + u_x(x,t)\,h + u_{xx}(x,t)\,\frac{h^2}{2} + \cdots$$

$$u(x-h,t) = u(x,t) + u_x(x,t)(-h) + u_{xx}(x,t)\,\frac{(-h)^2}{2} \cdots$$

Progresiva $\rightarrow$ $u_x(x,t) = \dfrac{u(x+h,t) - u(x,t)}{h}$

Regresiva $\rightarrow$ $u_x(x,t) = \dfrac{u(x-h,t) - u(x,t)}{-h}$

$$u_t(x,t) = \frac{u(x,t+k) - u(x,t)}{k}$$

$$u_t(x,t) = \frac{u(x,t-k) - u(x,t)}{-k}$$

Sumando

$$u(x+h,t) + u(x-h,t) = 2h(x,t) + u_{xx}(x,t)\,h^2$$

$$u_{xx}(x,t) = \frac{u(x+h,t) + u(x-h,t) - 2u(x,t)}{h^2}$$

$$u(x_2, t_4) \simeq V_{2_x}^{4_t}$$

$$u_t(x_2, t_5) = \frac{V_2^6 - V_2^5}{k}$$

$$u_t(x_e, t_n) = \frac{V_e^{n+1} - V_e^n}{k}$$

Método explícito

$$\rho c\, u_t - \tau u_{xx} = f(x,t)$$

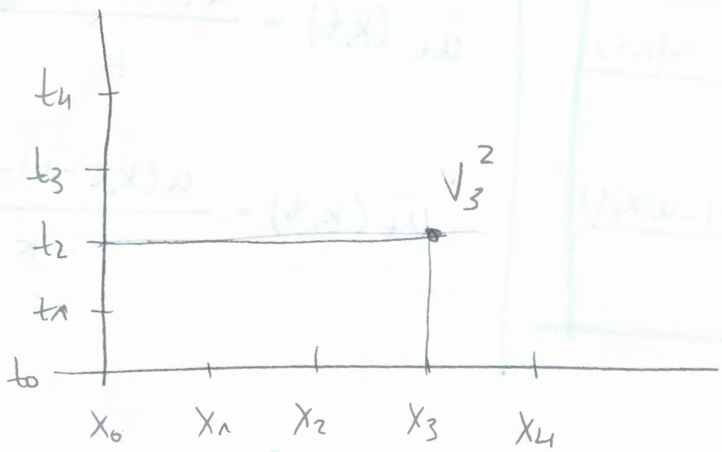
siempre es menos.

$$V_e^{n+1} = \lambda\left[V_{e+1}^n + V_{e-1}^n\right] + (1-2\lambda)V_e^n + \frac{k}{\rho c}\cdot f_e^n$$

$$\lambda = \frac{k\cdot\tau}{\rho c h^2}$$

Método implícito

$$(1+2\lambda)V_e^{n+1} - \lambda\left[V_{e+1}^{n+1} + V_{e-1}^{n+1}\right] = V_e^n + \frac{k}{\rho c}\cdot f_e^n$$



$$V_3^2 = \lambda\left[V_4^1 + V_2^1\right] + (1-2\lambda)V_3^1 + \frac{k}{\rho c}\cdot f_3^1$$

Máx, un sistema de 2 ecs. con 2 incógnitas

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$$4u_t - 3u_{xx} = 0 \qquad 0<x<1 \; ; \quad 0<t<10$$

$\boxed{f=0 \text{ cuando } n=0}$ ← enunciado.

$$\left.\begin{array}{l} u(0,t) = 10t \\ u_x(1,t) = 0 \end{array}\right\} \; 0<t<10$$

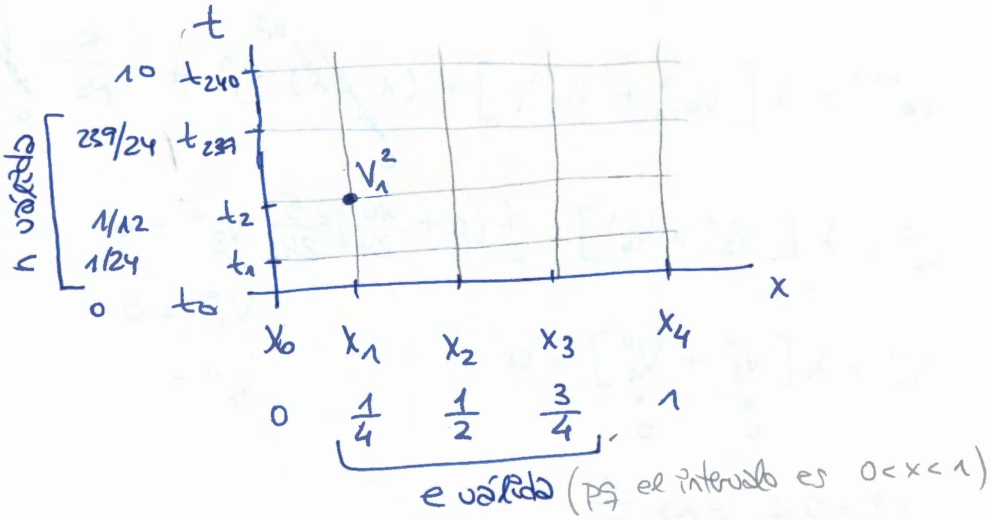
$$u(x,0) = 0 \qquad 0<x<1$$

M.D.F. explícito ¿ $u\left(\frac{1}{4}, \frac{1}{12}\right)$?

$h = \frac{1}{4}$

$k = \frac{1}{24}$

$k = \frac{1}{24}$ ↑ n



$h = \frac{1}{4} \to e$

$$\frac{1}{1/4} = 4 \; ; \quad e = 0, 1, 2, 3, 4$$

$$\frac{10}{1/24} = 240 \; ; \quad n = 0, 1, 2, \dots, 239, 240$$

$$V_e^{n+1} = \lambda \left[V_{e+1}^n + V_{e-1}^n \right] + (1-2\lambda) V_e^n + \frac{k}{\rho c} \cdot f_e^n$$

$$\left.\begin{array}{l} u(0,t) \simeq u(x_0, t_n) = V_0^n = 10 t_n \; ; \; \cancel{n=1,2 \dots 239} \\ u_x(1,t) \simeq u_x(x_4, t_n) = \dfrac{V_3^n - V_4^n}{-1/4} = 0 \end{array}\right\} n = 1, 2 \dots 239$$

$$\overset{u(x,0)}{\overbrace{u(1,0)}} \simeq u(x_e, t_0) = V_e^0 = 0 \; ; \quad e = 1, 2, 3 \quad \text{válidas.}$$

$$f(x,t) \simeq f(x_e, t_n) = f_e^n = 0 \quad \begin{cases} n = 1, 2, \dots 239 \\ e = 1, 2, 3 \end{cases}$$

$$\lambda = \frac{k \cdot \tau}{\rho c h^2} = \frac{\frac{1}{24} \cdot 3}{4 \cdot \frac{1}{16}} = 0{,}5$$

$V_0^n = 10 t_n$ ← siempre va "n"

$$\left. \begin{array}{l} V_0^n = 10 t_n \\ V_3^n = V_4^n \end{array} \right\} \quad n = 1, 2 \ldots, 239$$

$V_e^0 = 0 \;;\; e = 1, 2, 3$

$$f_e^n = 0 \quad \left| \begin{array}{l} n = 1, 2 \ldots 239 \\ e = 1, 2, 3 \end{array} \right.$$

$$V_e^{n+1} = \lambda \left[V_{e+1}^n + V_{e-1}^n \right] + (1 - \underbrace{\cancel{2\lambda}}_{0})^{0{,}5} \cdot V_e^n + \frac{k}{\rho c} \cdot \underbrace{\cancel{f_e^n}}_{0}$$

aplicamos método

$$\boxed{V_1^2} = \lambda \left[V_2^1 + V_0^1 \right] = \frac{1}{2} \left(0 + \frac{10}{24} \right) = \frac{5}{24}$$

$$V_2^1 = \lambda \left[\underset{0}{\underset{\shortparallel}{V_3^0}} + \underset{0}{\underset{\shortparallel}{V_1^0}} \right] = 0$$

$V_3^0 = 0$

$V_1^0 = 0$

$V_2^1 = 0$

$$V_0^1 = 10 t_1 = 10 \cdot \frac{1}{24}$$

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$$u_t - u_{xx} = \operatorname{sen} t \cdot \operatorname{sen} x \qquad 0 < x < \pi,\ t > 0$$

$$\left.\begin{array}{l} u_x(0,t) = 0 \\ u(\pi,t) = 0 \end{array}\right\} t > 0$$

$$u(x,0) = \operatorname{sen} x \ ; \ 0 < x < \pi$$

método difusión de calor

M.D.F. implícito

$$u\left(\frac{\pi}{3}, \frac{1}{3}\right), \left(\frac{2\pi}{3}, \frac{1}{3}\right), \left(\frac{\pi}{3}, \frac{2}{3}\right), \left(\frac{2\pi}{3}, \frac{2}{3}\right)$$

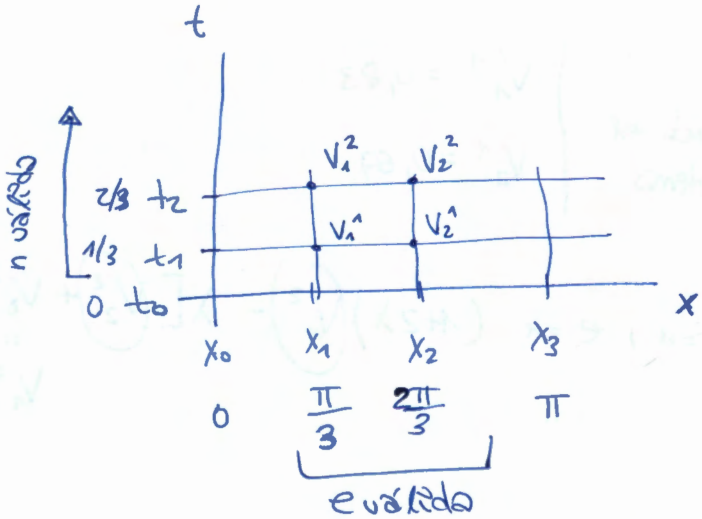
$$h = \frac{\pi}{3}$$

$$k = \frac{1}{3}$$

Dato: $f_e^0 = 0$

$$\frac{\pi}{\pi/3} = 3 \ ; \ e = 0,1,2,3$$

$$\frac{2/3}{1/3} = 2 \ ; \ n = 0,1,2$$



1. $u_x(0,t) = 0 \simeq u_x(x_0, t_n) = \dfrac{V_1^n - V_0^n}{\pi/3} = 0 \ ; \ n = 1,2,\dots \qquad V_1^n = V_0^n$

2. $u(\pi,t) \simeq u(x_3, t_n) = V_3^n = 0 \ ; \ n = 1,2\dots$

3. $u(x,0) \simeq u(x_e, t_0) = V_e^0 = \operatorname{sen} x_e \ ; \ e = 1,2$

4. $f(x,t) \simeq f(x_e, t_n) = f_e^n = \operatorname{sen} t_n \cdot \operatorname{sen} x_e \left\{ \begin{array}{l} e = 1,2 \\ n = 1,2\dots \end{array}\right.$

$$\lambda = \frac{3}{\pi^2}$$

Método implícito: $(1+2\lambda)\cdot V_e^{n+1} - \lambda\left[V_{e+1}^{n+1} + V_{e-1}^{n+1}\right] = V_e^n + \frac{k}{\rho c}\cdot f_e^n$

$n=0$, $e=1$ $\quad (1+2\lambda)\boxed{V_1^1} - \lambda\left[\boxed{V_2^1} + \underset{\substack{\shortparallel \\ V_1^1}}{V_0^1}\right] = \underset{\substack{\shortparallel \\ \text{sen}\frac{\pi}{3}}}{V_1^0} + \frac{1}{3}\underset{\substack{\shortparallel \\ 0}}{f_1^0}$

$e=2$ $\quad (1+2\lambda)V_2^1 - \lambda\left[\underset{\substack{\shortparallel \\ 0}}{V_3^1} + V_1^1\right] = \underset{\substack{\shortparallel \\ \text{sen}\frac{2\pi}{3}}}{V_2^0} + \frac{1}{3}\underset{\substack{\shortparallel \\ 0}}{f_2^0}$

Resolviendo el sistema $\left\{\begin{array}{l} V_1^1 = 0{,}83 \\ V_2^1 = 0{,}69 \end{array}\right.$

$n=1$, $e=1$ $\quad (1+2\lambda)V_1^2 - \lambda\left[V_2^2 + \underset{\substack{\shortparallel \\ V_1^2}}{V_0^2}\right] = \underset{\substack{\shortparallel \\ 0{,}83}}{V_1^1} + \frac{1}{3}\cdot\underset{\substack{\shortparallel \\ \text{sen}\frac{1}{3}\cdot\text{sen}\frac{\pi}{3}}}{f_1^1}$

$e=2$ $\quad (1+2\lambda)V_2^2 - \lambda\left[\underset{\substack{\shortparallel \\ 0}}{V_3^2} + V_1^2\right] = \underset{\substack{\shortparallel \\ 0{,}69}}{V_2^1} + \frac{1}{3}\cdot\underset{\substack{\shortparallel \\ \text{sen}\frac{1}{3}\cdot\text{sen}\frac{2\pi}{3}}}{f_2^1}$

Resolviendo $\left\{\begin{array}{l} V_1^2 = 0{,}86 \\ V_2^2 = 0{,}65 \end{array}\right.$

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$$u_t - u_{xx} = 5\pi + 3 \qquad 0<x<\pi \;,\; t>0$$

$$\left.\begin{aligned} u_x(0,t) &= 10 \\ u_x(\pi,t) &= t+1 \end{aligned}\right\} t>0$$

$$u(x,0) = \cos x \qquad 0<x<\pi$$

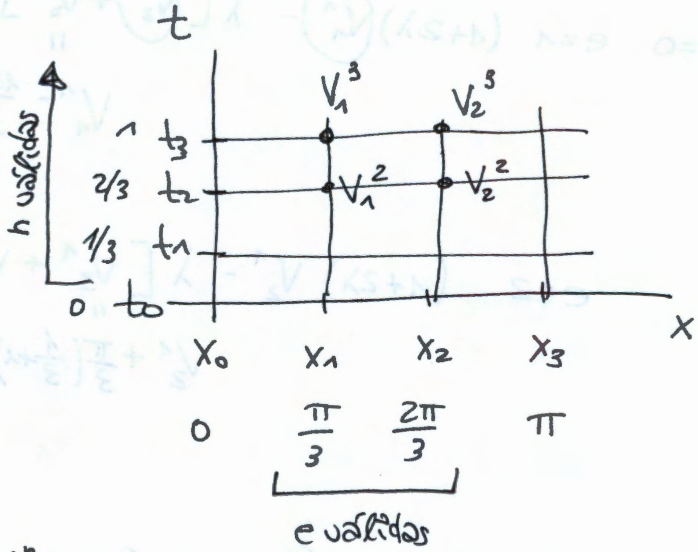
$$u(x,t) = \left\{ \left(\frac{\pi}{3}, \frac{2}{3}\right), \left(\frac{2\pi}{3}, \frac{2}{3}\right), \left(\frac{\pi}{3}, 1\right), \left(\frac{2\pi}{3}, 1\right) \right\}$$

M.D.F.: implícito

$h = \frac{\pi}{3}$ Dato: $f_e^0 = f_e^n \longrightarrow$ se puede usar f para $n=0$.

$k = \frac{1}{3}$

$$\frac{\pi}{\pi/3} = 3 \;;\; e = 0,1,2,3$$



$$\left.\begin{aligned} u_x(0,t) &\simeq u_x(x_0,t_n) = \frac{V_1^n - V_0^n}{\pi/3} = 10 \;;\; n = 1,2\ldots \\ u_x(\pi,t) &\simeq u_x(x_3,t_n) = \frac{V_2^n - V_3^n}{-\pi/3} = t_n + 1 . \end{aligned}\right\} n=1,2$$

$$u(x,0) \simeq u(x_e,t_0) = V_e^0 = \cos x_e \;,\; e = 1,2$$

$$f(x,t) \simeq f(x_e,t_n) = f_e^n = 5\pi + 3 \left\{ \begin{aligned} &e = 1,2 \\ &n = 1,2\ldots \end{aligned}\right.$$

$$\lambda = 0{,}304 = \frac{k\cdot\tau}{\rho c\cdot h^2}$$

$$\frac{k}{\rho c} = \frac{1}{3}$$

$$\left.\begin{aligned} V_1^n - V_0^n &= \frac{\pi}{3}\cdot 10 \\ V_2^n - V_3^n &= -\frac{\pi}{3}(t_n+1) \end{aligned}\right\} \quad n = 1,2\ldots$$

$$V_e^0 = \cos x_e \quad ; \quad e = 1,2$$

$$f_e^n = 5\pi + 3 \quad \left\{\begin{aligned} & e = 1,2 \\ & n = 1,2\ldots \end{aligned}\right.$$

$$f_e^0 = f_e^n$$

$$n=0 \quad e=1 \quad (1+2\lambda)V_1^1 - \lambda\Big[V_2^1 + \underset{\substack{\parallel \\ V_1^1 - \frac{10\pi}{3}}}{V_0^1}\Big] = \overset{\substack{\cos\pi/3 \\ \parallel}}{V_1^0} + \frac{1}{3}\underset{\substack{\parallel \\ 5\pi+3}}{f_1^0}$$

$$e=2 \quad (1+2\lambda)V_2^1 - \lambda\Big[\underset{\substack{\parallel \\ V_2^1 + \frac{\pi}{3}\left(\underset{\substack{\parallel \\ t_1}}{\frac{1}{3}}+1\right)}}{V_3^1} + V_1^1\Big] = \underset{\substack{\parallel \\ \cos\frac{2\pi}{3}}}{V_2^0} + \frac{1}{3}\cdot\underset{\substack{\parallel \\ 5\pi+3}}{f_2^0}$$

$$\left\{\begin{aligned} V_1^1 &= 4{,}02 \\ V_2^1 &= 5{,}64 \end{aligned}\right.$$

$$n=1 \quad e=1 \quad (1+2\lambda)V_1^2 - \lambda\Big[V_2^2 + \underset{\substack{\parallel \\ V_1^2 - \frac{10\pi}{3}}}{V_0^2}\Big] = \underset{\substack{\parallel \\ 4{,}02}}{V_1^1} + \frac{1}{3}\underset{\substack{\parallel \\ 5\pi+3}}{f_1^1}$$

$$e=2 \quad (1+2\lambda)V_2^2 - \lambda\Big[\underset{\substack{\parallel \\ V_2^2 + \frac{\pi}{3}\left(\frac{2}{3}+1\right)}}{V_3^2} + V_1^2\Big] = \underset{\substack{\parallel \\ 5{,}64}}{V_2^1} + \frac{1}{3}\cdot\underset{\substack{\parallel \\ 5\pi+3}}{f_2^1}$$

$$\left\{\begin{aligned} V_1^2 &= 8{,}03 \\ V_2^2 &= 11{,}34 \end{aligned}\right.$$

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$n=2 \quad e=1 \qquad (1+2\lambda)V_1^3 - \lambda\left[V_2^3 + V_0^3\right] = V_1^2 + \frac{1}{3} f_1^2$

$V_0^3 = V_1^3 - \frac{10}{3}\pi \qquad V_1^2 = 8{,}03 \qquad f_1^2 = 5\pi + 3$

$V_1^3 = 12{,}35$

$V_2^3 = 16{,}77$

$e=2 \qquad (1+2\lambda)V_2^3 - \lambda\left[V_3^3 + V_1^3\right] = V_2^2 + \frac{1}{3} f_2^2$

$V_3^3 = V_2^3 + \frac{\pi}{3}(1+1) \qquad V_2^2 = 11{,}34 \qquad f_2^2 = 5\pi + 3$

$\boxed{11}$

$u_t - u_{xx} = \operatorname{sen} x \qquad -\pi < x < \pi \quad , t > 0$

$\left.\begin{array}{l} u(-\pi,t) = t^2 \\ u_x(\pi,t) = 0 \end{array}\right\} \; t > 0$

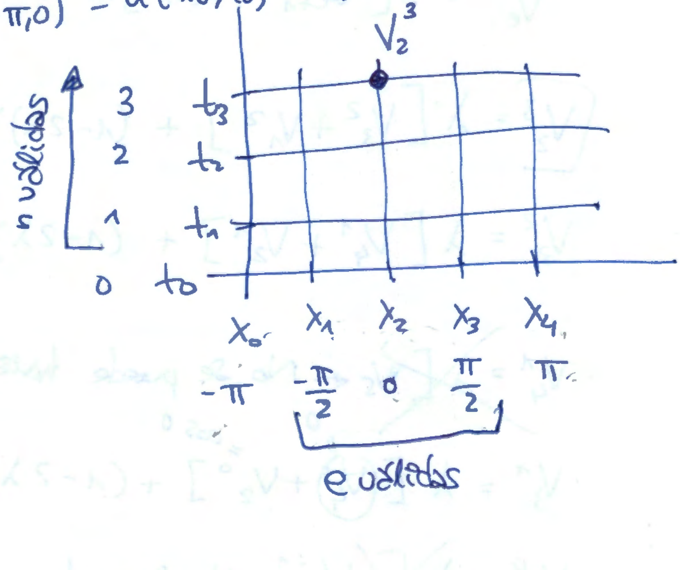
$u(x,0) = \cos x \; ; \; -\pi < x < \pi$

M.D.F. explícito ¿$u(0,3)$?

$h = \frac{\pi}{2}$

$k = 1$

Datos: $\left\{\begin{array}{l} f(x,0) = \operatorname{sen} x \quad , \; -\pi < x < \pi \\ u(\pi,0) = 0 \longrightarrow u(\pi,0) \simeq u(x_4,t_0) = V_4^0 = 0 \\ u(-\pi,0) = 0 \qquad u(-\pi,0) \simeq u(x_0,t_0) = V_0^0 = 0 \end{array}\right.$



$\frac{2\pi}{\pi/2} = 4 \; ; \; e = 1,2,3,4$

$\frac{3}{1} = 3 \; ; \; n = 0,1,2,3$

$$u(-\pi,t) \simeq u(x_0,t_n) = V_0^n = t_n^2$$
$$u_x(\pi,t) \simeq u_x(x_4,t_n) = \frac{V_3^n - V_4^n}{-\pi/2} = 0$$
$n = 1,2\ldots$

$$u(x,0) \simeq u(x_e,t_0) = V_e^0 = \cos x_e \qquad e = 1,2,3$$

$$f(x,t) \simeq f(x_e,t_n) = f_e^n = \operatorname{sen} x_e \;; \quad \begin{cases} e = 1,2,3 \\ n = 1,2\ldots \end{cases}$$

$$\lambda = 0{,}405$$

$$\frac{k}{\rho c} = 1$$

$$f(x,0) = f(x_e,t_0) = f_e^0 = \operatorname{sen} x_e \;; \quad e = 1,2,3$$

$$\left.\begin{aligned} V_0^n &= t_n^2 \\ V_3^n &= V_4^n \end{aligned}\right\} n = 1,2\ldots$$

$$V_e^0 = \cos x_e \;; \quad e = 1,2,3$$

$$f_e^n = \operatorname{sen} x_e \quad \begin{cases} e = 1,2,3 \\ n = 1,2\ldots \end{cases}$$

$$f_e^0 = \operatorname{sen} x_e \;, \quad e = 1,2,3$$

$$V_e^{n+1} = \lambda\left[V_{e+1}^n + V_{e-1}^n\right] + (1-2\lambda)V_e^n + \frac{k}{\rho c}\cdot f_e^n$$

$$\boxed{V_2^3} = \lambda\left[V_3^2 + V_1^2\right] + (1-2\lambda)V_2^2 + f_2^2 = \boxed{0{,}59}$$ El que pide el enunciado.

$$V_3^2 = \lambda\left[V_4^1 + V_2^1\right] + (1-2\lambda)V_3^1 + f_3^1 = 1{,}91$$

$V_4^1 = \cancel{\lambda\,[\,V_5}$ ← No se puede hallar.

$$V_3^1 = \lambda\left[\underset{\parallel}{(V_4^0)}^{0} + \underbrace{V_2^0}_{=\cos 0}\right] + (1-2\lambda)\underset{\parallel}{V_3^0}^{\cos\frac{\pi}{2}} + \underset{\parallel}{f_3^0}_{\operatorname{sen}\frac{\pi}{2}} = 1{,}405$$

$V_4^0 = \cancel{\lambda\,[\,V_5^{-1}}$ ← No se puede

20.09.2016

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$V_4^0 = 0$

$V_2^0 = 1$

$V_3^0 = 0$

$f_3^0 = 1$

$V_3^1 = 1,405$

$V_4^1 = 1,405$

$f_2^0 = 0$

$V_2^1 = 0,19$

$V_3^2 = 1,91$

$V_0^1 = 1$

$V_0^0 = 0$ (dato enunciado)

$f_1^0 = -1$

$V_1^1 = -0,59$

$V_1^2 = -0,63$

$$V_2^1 = \lambda [V_3^0 + \underset{\underset{\cos\frac{-\pi}{2}}{\shortparallel}}{V_1^0}] + (1-2\lambda) V_2^0 + \underset{\underset{\text{sen } 0}{\shortparallel}}{f_2^0} = 0,19$$

$$V_1^2 = \lambda [V_2^1 + V_0^1] + (1-2\lambda) V_1^1 + f_1^1 = -0,63$$

$$V_0^1 = t_1^2$$

$$V_1^1 = \lambda [V_2^0 + \underset{\underset{0}{\shortparallel}}{V_0^0}] + (1-2\lambda) V_1^0 + \underset{\underset{\text{sen } -\frac{\pi}{2}}{\shortparallel}}{f_1^0} = -0,59$$

$$V_2^2 = \lambda [V_3^1 + V_1^1] + (1-2\lambda) V_2^1 + f_2^1 = 0,36$$

26.09.2016

11 BIS.

$u_t - u_{xx} = \text{sen } x \qquad -\pi < x < \pi \qquad t > 0$

$\left.\begin{array}{l} u(-\pi,t) = t^2 \\ u_x(\pi,t) = 0 \end{array}\right\} t > 0$

$u(x,0) = \cos x \qquad -\pi < x < \pi$

Datos:

$f(x,0) = \text{sen } x; \; -\pi < x < \pi$

$u(-\pi,0) = 0$

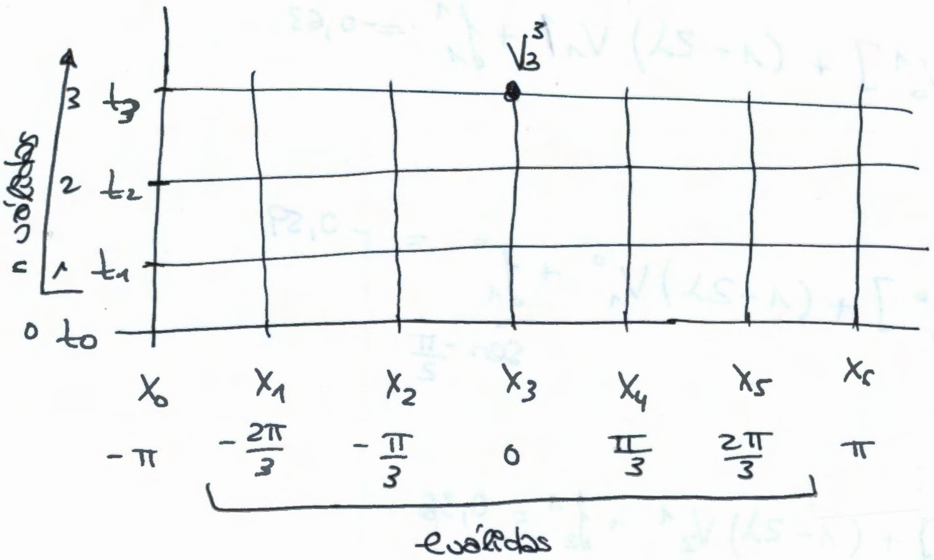
$u(\pi,0) = 0$

M.D.F. explícito ¿$u(0,3)$?

$h = \frac{\pi}{3}$; $k = 1$

$\frac{2\pi}{\pi/3} = 6$; $e = 0,1,2,3,4,5,6$

$\frac{3}{1} = 3$; $n = 0,1,2,3$



$\left.\begin{array}{l} u(-\pi,t) \simeq u(x_0,t_n) = V_0^n = t_n^2 \; ; \quad n = 1,2\ldots \\ u_x(\pi,t) \simeq u_x(x_6,t_n) = \dfrac{V_5^n - V_6^n}{-\pi/3} = 0 \end{array}\right\} n = 1,2\ldots$

$u(x,0) \simeq u(x_e,t_0) = V_e^0 = \cos x_e$; $e = 1, - 5$

$f(x,t) \simeq f(x_e,t_n) = \boxed{f_e^n = \text{sen } x_e} \left\{ \begin{array}{l} e = 1 \ldots 5 \\ n = 1,2\ldots \end{array}\right.$

$f(x,0) = \text{sen } x; \; -\pi < x < \pi \longrightarrow f(x,0) \simeq f(x_e,t_0) = \boxed{f_e^0 = \text{sen } x_e}$; $e = 1..5$

$u(-\pi,0) = 0 \longrightarrow u(-\pi,0) \simeq u(x_0,t_0) = V_0^0 = 0$

$u(\pi,0) = 0 \longrightarrow u(\pi,0) \simeq u(x_6,t_0) = V_6^0 = 0$

$\lambda = 0{,}91$

$$V_e^{n+1} = \lambda \left[V_{e+1}^n + V_{e-1}^n \right] + (1-2\lambda) V_e^n + \frac{k}{\rho c} \cdot f_e^1$$

$\dfrac{k}{\rho c} = 1$

$\left.\begin{array}{l} V_0^n = t_n^2 \\ V_5^n = V_6^n \end{array}\right\} n = 1, 2 \ldots$

$V_e^0 = \cos x_e \quad ; e = 1 - 5$

$f_e^n = \operatorname{sen} x_e \left\{ \begin{array}{l} e = 1 - 5 \\ n = 0, 1, 2 \ldots \end{array}\right.$

↑ dato

$V_0^0 = V_6^0 = 0$

$V_6^0 = 0$

$V_4^0 = 0{,}5$

$V_5^0 = -0{,}5$

$f_5^0 = 0{,}87$

$V_5^1 = 1{,}74$

$V_2^0 = 0{,}5$

$V_3^0 = 1$

$f_3^0 = 0$

$V_3^1 = 0{,}08$

$f_4^0 = 0{,}87$

$V_4^1 = 0{,}91$

$V_4^2 = 1{,}77$

$V_1^0 = -0{,}5$

$f_1^0 = -0{,}87$

$V_1^1 = 0$

$V_2^1 = -0{,}83$

$f_2^0 = -0{,}87$

$V_3^3 = \lambda \left[V_4^2 + V_2^2 \right] + (1-2\lambda) V_3^2 + f_3^2 = 1{,}51$

$V_4^2 = \lambda \left[V_5^1 + V_3^1 \right] + (1-2\lambda) V_4^1 + \underset{\overset{\shortparallel}{0{,}87}}{f_4^1} = 1{,}77$

$V_5^1 = \lambda \left[\underset{\overset{\shortparallel}{0}}{V_6^0} + \underset{\overset{\shortparallel}{\cos\frac{\pi}{3}}}{V_4^0} \right] + (1-2\lambda) \underset{\overset{\shortparallel}{\cos\frac{2\pi}{3}}}{V_5^0} + \underset{\overset{\shortparallel}{\operatorname{sen}\frac{2\pi}{3}}}{f_5^0} = 1{,}74$

$V_3^1 = \lambda \left[\underset{\overset{\shortparallel}{0{,}5}}{V_4^0} + \underset{\overset{\shortparallel}{\cos(-\frac{\pi}{3})}}{V_2^0} \right] + (1-2\lambda) \underset{\overset{\shortparallel}{\cos 0}}{V_3^0} + \underset{\overset{\shortparallel}{\operatorname{sen} 0}}{f_3^0} = 0{,}08$

$V_4^1 = \lambda \left[V_5^0 + V_3^0 \right] + (1-2\lambda) V_4^0 + \underset{\overset{\shortparallel}{\operatorname{sen}\frac{\pi}{3}}}{f_4^0} = 0{,}91$

$V_2^2 = \lambda \left[V_3^1 + V_1^1 \right] + (1-2\lambda) V_2^1 + f_2^1 = -0{,}11$

$V_1^1 = \lambda \left[V_2^0 + \underset{\overset{\shortparallel}{0}}{V_0^0} \right] + (1-2\lambda) \underset{\overset{\shortparallel}{\cos -\frac{2\pi}{3}}}{V_1^0} + \underset{\overset{\shortparallel}{\operatorname{sen} -\frac{2\pi}{3}}}{f_1^0} = 0$

$V_2^1 = \lambda \left[V_3^0 + V_1^0 \right] + (1-2\lambda) V_2^0 + \underset{\overset{\shortparallel}{\operatorname{sen} -\frac{\pi}{3}}}{f_2^0} = -0{,}83$

$V_3^2 = \lambda \left[V_4^1 + V_2^1 \right] + (1-2\lambda) V_3^1 + f_3^1 = 0$

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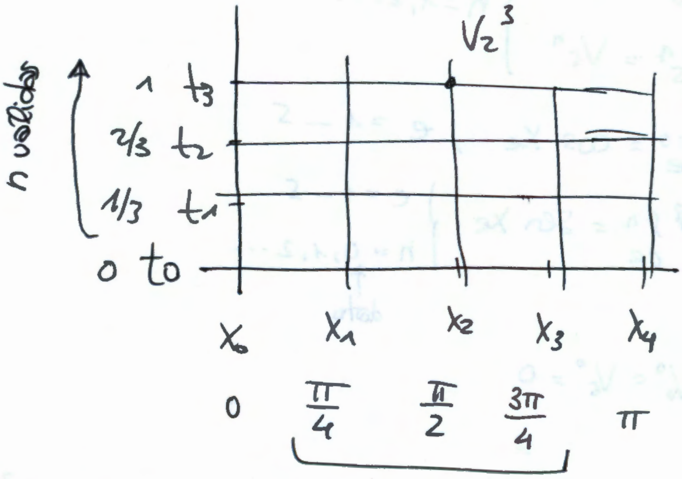
27.09.2016

$$u_t - 2u_{xx} = x - \pi \qquad 0 < x < \pi;\ t > 0$$

$$\left.\begin{aligned} u_x(0,t) &= t \\ u(\pi,t) &= 0 \end{aligned}\right\} t > 0$$

$$u(x,0) = \text{sen } 5x \qquad 0 < x < \pi$$

MDF expl. ¿$u(\frac{\pi}{2}, 1)$?



Datos: $f_e^n = f_e^0$

$h = \frac{\pi}{4}$

$k = \frac{1}{3}$

$u(\pi,0) = 0$

$u(0,0) = 0$

$\frac{\pi}{\pi/4} = 4$; $e = 0,1,2,3,4$

$\frac{1}{1/3} = 3$; $n = 0,1,2,3$

$\lambda = 1{,}08$

$\frac{k}{h^2} = \frac{1}{3}$

$$\left.\begin{aligned} u_x(0,t) &\simeq u_x(x_0,t_n) = \frac{V_1^n - V_0^n}{\pi/4} = t_n \\ u(\pi,t) &\simeq u(x_4,t_n) = V_4^n = 0 \end{aligned}\right\} n = 1,2\ldots$$

$$u(x,0) \simeq u(x_e, t_0) = V_e^0 = \text{sen } 5x_e \ ; \ e = 1,2,3$$

$$f(x,t) \simeq f(x_e,t_n) = f_e^n = x_e - \pi \quad \begin{cases} e = 1,2,3 \\ n = 1,2\ldots \end{cases}$$

$u(\pi,0) = 0$; $\rightarrow u(\pi,0) \simeq u(x_4,t_0) = V_4^0 = 0$

$u(0,0) = 0 \rightarrow u(0,0) \simeq u(x_0,t_0) = V_0^0 = 0$

$$\circledast \; \left.\begin{array}{l} V_1^n - V_0^n = \frac{\pi}{4} t_n \\ V_4^n = 0 \end{array}\right\} n = 1,2\ldots$$

$V_e^0 = \text{sen}\, 5x_e \; ; \; e = 1,2,3$

$$f_e^n = x_e - \pi \quad \left\{\begin{array}{l} e = 1,2,3 \\ n = 0,1,2\ldots \end{array}\right.$$

$f_e^n = f_e^0$

$V_3^0 = 0$

$V_4^0 = 0$

$$V_e^{n+1} = \lambda\left[V_{e+1}^n + V_{e-1}^n\right] + (1-2\lambda)V_e^n + \frac{k}{\rho c}\cdot f_e^n$$

$$V_2^3 = \lambda\left[V_3^2 + V_1^2\right] + (1-2\lambda)V_2^2 + \frac{1}{3}\cdot f_2^2 = -18{,}76$$

$$V_3^2 = \lambda\left[\underset{0}{\underset{\shortparallel}{V_4^1}} + V_2^1\right] + (1-2\lambda)V_3^1 + \frac{1}{3}\cdot \underset{-\frac{\pi}{4}}{\underset{\shortparallel}{f_3^1}} = -5{,}64$$

$$V_2^1 = \lambda\left[\underset{\text{sen}\frac{3\pi}{4}}{\underset{\shortparallel}{V_3^0}} + \underset{\text{sen}\frac{5\pi}{4}}{\underset{\shortparallel}{V_1^0}}\right] + (1-2\lambda)\underset{\text{sen}\frac{5\pi}{2}}{\underset{\shortparallel}{V_2^0}} + \frac{1}{3}\cdot \underset{\frac{\pi}{2}-\pi}{\underset{\shortparallel}{f_2^0}} = -3{,}22$$

$$V_3^1 = \lambda\left[V_4^0 + V_2^0\right] + (1-2\lambda)V_3^0 + \frac{1}{3}\underset{\frac{3\pi}{4}-\pi}{\underset{\shortparallel}{f_3^0}} = 1{,}64$$

$$V_1^2 = \lambda\left[V_2^1 + V_0^1\right] + (1-2\lambda)\cdot V_1^1 + \frac{1}{3}\cdot f_1^1 = -4{,}63$$

$$V_0^1 = \lambda\left[V_1^0 + \underset{¿?}{\underset{\shortparallel}{V_4^0}}\right.$$

$$V_1^1 = \lambda\left[V_2^0 + V_0^0\right] + (1-2\lambda)V_1^0 + \frac{1}{3}\cdot \underset{\frac{\pi}{4}-\pi}{\underset{\shortparallel}{f_1^0}} = 1{,}11$$

$$\circledast \; 1{,}11 - V_0^1 = \frac{\pi}{4}\cdot\frac{1}{3} \rightarrow V_0^1 = 0{,}84$$

$V_4^1 = 0$

$V_3^0 = -0{,}71$

$V_1^0 = -0{,}71$

$V_2^0 = 1$

$f_2^0 = -\frac{\pi}{2}$

$V_2^1 = -3{,}22$

$V_4^0 = 0$

$f_3^0 = -\frac{\pi}{4}$

$V_3^1 = 1{,}64$

$V_3^2 = -5{,}64$

$V_0^0 = 0$

$f_1^0 = -\frac{3}{4}\pi$

$V_1^1 = 1{,}11$

$V_0^1 = 0{,}84$

$V_1^2 = -4{,}63$

$V_2^2 = 6{,}16$

$V_2^3 = -18{,}76$.

$$V_2^2 = \lambda \left[V_3^1 + V_1^1 \right] + (1-2\lambda) V_2^1 + \frac{1}{3} \cdot f_2^1 = 6,16$$

$\boxed{14}$

$$U_t - 0,15 \cdot U_{xx} = 2xt \qquad 0<x<1 \; ; \; t>0$$

$$\left. \begin{aligned} u(0,t) &= 1 \\ u(1,t) &= t^2 \end{aligned} \right\} t>0$$

$$\frac{1}{1/3} = 3 \; ; \; e = 0,1,2,3$$

$$\frac{1}{1/3} = 3 \; ; \; n = 0,1,2,3$$

$$u(x,0) = \cos \pi x + 1 - x \qquad 0<x<1$$

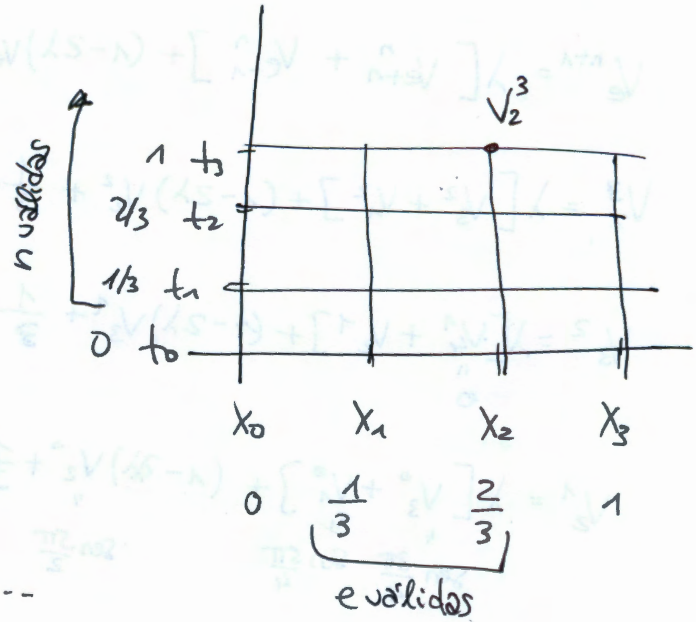
MDF expl. ¿$u(\frac{2}{3}, 1)$?

$$h = \frac{1}{3} \; , \; k = \frac{1}{3}$$

Datos: $f(x,0) = 2x \; ; \; 0<x<1$

$u(1,0) = 1$

$u(0,0) = -1$



$$u(0,t) \simeq u(X_0, t_n) = V_0^n = 1 \; ; \; n = 1,2 \ldots$$

$$u(1,t) \simeq u(X_3, t_n) = V_3^n = t_n^2$$

$$u(x,0) \simeq u(X_e, t_0) = V_e^0 = \cos \pi X_e + 1 - X_e \; ; \; e = 1,2$$

$$f(x,t) \simeq f(X_e, t_n) = f_e^n = 2 X_e t_n \left\{ \begin{aligned} e &= 1,2 \\ n &= 1,2 \ldots \end{aligned} \right.$$

$$f(x,0) \simeq f(X_e, t_0) = f_e^0 = 2X_e \; ; \; e = 1,2$$

$$u(1,0) \simeq u(X_3, t_0) = V_3^0 = 1$$

$$u(0,0) \simeq u(X_0, t_0) = V_0^0 = -1$$

$$\left.\begin{array}{l} V_0^n = 1 \\ V_3^n = t_n^2 \end{array}\right\} \quad n = 1, 2 \ldots$$

$$V_e^0 = \cos \pi \cdot x_e + 1 - x_e \ ; \ e = 1, 2$$

$$f_e^n = 2x_e \cdot t_n \quad \left\{\begin{array}{l} e = 1, 2 \\ n = 1, 2 \ldots \end{array}\right.$$

$$f_e^0 = 2x_e \ ; \ e = 1, 2$$

$$V_0^0 = -1$$

$$V_3^0 = 1$$

$$V_e^{n+1} = \lambda \left[\underbrace{V_3^2}_{= \left(\frac{2}{3}\right)^2} + V_1^2 \right] + (1 - 2\lambda) \cdot V_2^2 + \frac{1}{3} \cdot f_2^2$$

$$V_1^2 = \lambda \left[V_2^1 + V_0^1 \right] + (1 - 2\lambda) V_1^1 + \frac{1}{3} \cdot f_1^1 = 1{,}13$$

$$V_2^1 = \lambda \left[\underbrace{V_3^0}_{= 1} + \underbrace{V_1^0}_{= \cos\frac{\pi}{3} + 1 - \frac{1}{3}} \right] + (1 - 2\lambda) \underbrace{V_2^0}_{= \cos\frac{2\pi}{3} + 1 - \frac{2}{3}} + \frac{1}{3} \underbrace{f_2^0}_{= 2 \cdot \frac{2}{3}} = 1{,}39$$

$$V_1^1 = \lambda \left[V_2^0 + \underbrace{V_0^0}_{= -1} \right] + (1 - 2\lambda) V_1^0 + \frac{1}{3} \cdot \underbrace{f_1^0}_{= 2/3} = -0{,}18$$

$$V_2^2 = \lambda \left[\underbrace{V_3^1}_{= (1/3)^2} + V_1^1 \right] + (1 - 2\lambda) V_2^1 + \frac{1}{3} \cdot \underbrace{f_2^1}_{= 2 \cdot \frac{2}{3} \cdot \frac{1}{3}} = 0{,}25$$

$$V_2^3 = \lambda \left[\underbrace{V_3^2}_{= \left(\frac{2}{3}\right)^2} + V_1^2 \right] + (1 - 2\lambda) V_2^2 + \frac{1}{3} \cdot \underbrace{f_2^2}_{= 2 \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{9}} = 1{,}03$$

$V_3^2 = 0{,}44$

$V_1^0 = 1{,}16$

$V_2^0 = -0{,}17$

$f_2^0 = \frac{4}{3}$

$V_2^1 = 1{,}39$

$V_0^1 = 1$

$f_1^0 = 2/3$

$V_1^1 = -0{,}18$

$f_1^1 = \frac{2}{9}$

$V_1^2 = 1{,}13$

$V_2^2 = 1/9$

$f_2^1 = \frac{4}{9}$

$V_2^2 = 0{,}25$

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$$u_t - u_{xx} = \operatorname{sen} x \qquad -\pi < x < 2\pi \ ; \ t>0$$

$$\left.\begin{aligned} u(-\pi,t) &= t^2 \\ u_x(2\pi,t) &= 0 \end{aligned}\right\} t>0$$

$$u(x,0) = \cos x \qquad -\pi < x < 2\pi$$

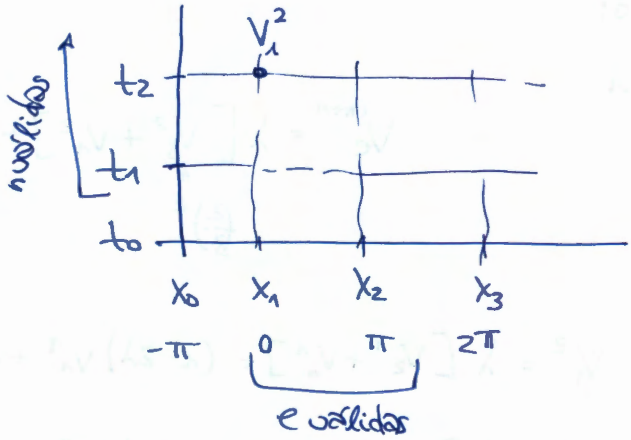
M.D.F. implícito ¿$u(0,2)$?

$h=\pi$, $k=1$

Dato: $f_e^0 = f_e^n$

$\frac{3\pi}{\pi} = 3$; $e = 0,1,2,3$

$\frac{2}{1} = 2$; $n = 0,1,2$



$u(-\pi,t) \simeq u(x_0,t_n) = V_0^n = t_n^2$; $n = 1,2\ldots$

$u_x(2\pi,t) \simeq u_x(x_3,t_n) = \frac{V_2^n - V_3^n}{-\pi} = 0$

$u(x,0) \simeq u(x_e,t_0) = V_e^0 = \cos x_e$; $e = 1,2$

$f(x,t) \simeq f(x_e,t_n) = f_e^n = \operatorname{sen} x_e \quad \begin{cases} e = 1,2 \\ n = 1,2\ldots \end{cases}$

$\lambda = 0{,}101$

$\frac{k}{h} = 1$

$$\left.\begin{aligned} V_0^n &= t_n^2 \\ V_2^n &= V_3^n \end{aligned}\right\} n = 1,2\ldots$$

$V_e^0 = \cos x_e$, $e = 1,2$

$f_e^n = \operatorname{sen} x_e \quad \begin{cases} e = 1,2 \\ n = 0,1,2\ldots \end{cases}$

$f_e^0 = f_e^n$

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$$(1+2\lambda) V_e^{n+1} - \lambda \left[V_{e+1}^{n+1} + V_{e-1}^{n+1} \right] = V_e^n + \frac{k}{\rho c} \cdot f_e^n$$

$n=0$ $e=1$ $(1+2\lambda) V_1^1 - \lambda [V_2^1 + \underset{1^2}{V_0^1}] = \underset{\cos 0}{V_1^0} + \underset{\text{sen } 0}{f_1^0}$

$e=2$ $(1+2\lambda) V_2^1 - \lambda [\underset{V_2^1}{V_3^1} + V_1^1] = \underset{\cos \pi}{V_2^0} + \underset{\text{sen } \pi}{f_2^0}$

$V_1^1 = 0{,}85$

$V_2^1 = -0{,}83$

$V_1^2 = 0{,}98$

$V_2^2 = -0{,}66$

$n=1$ $e=1$ $(1+2\lambda) V_1^2 - \lambda [V_2^2 + \underset{2^2}{V_0^2}] = \underset{0{,}85}{V_1^1} + \underset{\text{sen } 0}{f_1^1}$

$e=2$ $(1+2\lambda) V_2^2 - \lambda [\underset{V_2^2}{V_3^2} + V_1^2] = \underset{-0{,}83}{V_2^1} + \underset{\text{sen } \pi}{f_2^1}$

PROBLEMA DE EXAMEN

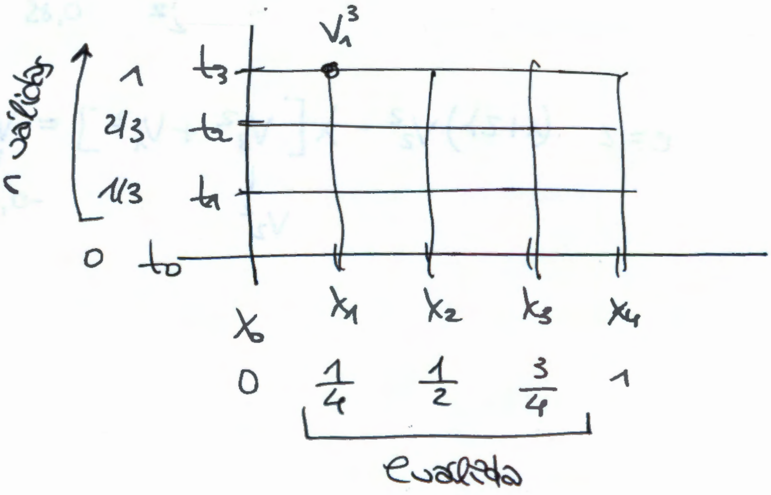
$$25u_t - u_{xx} = t^2 + x + 1 \qquad 0<x<1\ ;\ t>0$$

$$\left.\begin{array}{l} u(0,t) = \dfrac{t^2}{2} \\ u(1,t) = 0 \end{array}\right\} t>0$$

$$u(x,0) = x^2 \quad 0<x<1$$

MDF expl. para hallar ¿$u\left(\frac{1}{4},1\right)$?

$h = \frac{1}{4}$; $k = \frac{1}{3}$



Datos: $f(x,0) = 0$

$u(0,0) = h$

$u(1,0) = 0$

$\frac{1}{1/4} = 4$; $e = 0,1,2,3,4$

$\frac{1}{1/3} = 3$; $n = 0,1,2,3$

$u(0,t) \simeq u(x_0,t_n) = V_0^n = \frac{t_n^2}{2}$; $n = 1,2\ldots$

$u(1,t) \simeq u(x_4,t_n) = V_4^n = 0$

$u(x,0) \simeq u(x_e,t_0) = V_e^0 = x_e^2$; $e = 1,2,3$

$f(x,t) \simeq f(x_e,t_n) = f_e^n = t_n^2 + x_e + 1 \quad \left\{\begin{array}{l} e = 1,2,3 \\ n = 1,2\ldots \end{array}\right.$

$f(x,0) = 0 \rightarrow f(x,0) \simeq f(x_e,t_0) = f_e^0 = 0$; $e = 1,2,3$

$u(0,0) = h \rightarrow u(0,0) \simeq u(x_0,t_0) = V_0^0 = 1/4$

$u(1,0) = 0 \rightarrow u(1,0) \simeq u(x_4,t_n) = V_4^n = 0$

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$$\left.\begin{aligned} V_0^n &= \frac{t_n^2}{2} \\ V_4^n &= 0 \end{aligned}\right\} n = 1, 2 \ldots$$

$\lambda = 0{,}213$

$\frac{k}{\rho c} = \frac{1}{25}$

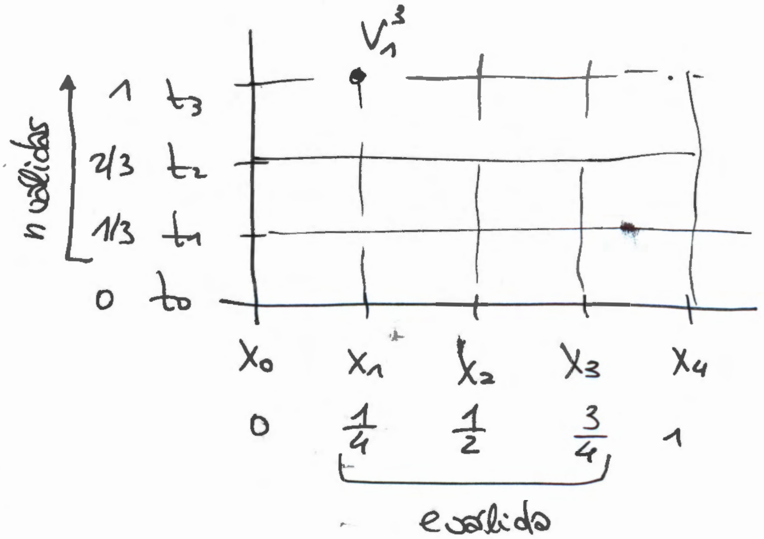
$V_e^0 = x_e^2 \; ; \; e = 1,2,3$

$f_e^n = t_n^2 + x_e + 1 \quad \left\{ \begin{aligned} e &= 1,2,3 \\ n &= 1,2 \ldots \end{aligned} \right.$

$f_e^0 = 0 \; ; \quad e = 1,2,3$

$V_0^0 = \frac{1}{4}$

$V_4^0 = 0$



$V_e^{n+1} = \lambda \left[V_{e+1}^n + V_{e-1}^n \right] + (1 - 2\lambda) V_e^n + \frac{k}{\rho c} \cdot f_e^n$

$V_1^3 = \lambda \left[V_2^2 + V_0^2 \right] + (1-2\lambda) V_1^2 + \frac{1}{25} \cdot \underbrace{f_1^2}_{\left(\frac{2}{3}\right)^2 + \frac{1}{4} + 1} = 0{,}23$

$V_2^2 = \lambda \left[V_3^1 + V_1^1 \right] + (1-2\lambda) V_2^1 + \frac{1}{25} \cdot \underbrace{f_2^1}_{(1/4)^2 + \frac{1}{2} + 1} = 0{,}29$

$V_3^1 = \lambda \left[V_4^0 + \underbrace{V_2^0}_{1/4} \right] + (1-2\lambda) \underbrace{V_3^0}_{\left(\frac{3}{4}\right)^2} + \frac{1}{25} \cdot \underbrace{f_3^0}_{0} = 0{,}37$

$V_1^1 = \lambda \left[V_2^0 + V_0^0 \right] + (1-2\lambda) \underbrace{V_1^0}_{(1/4)^2} + \frac{1}{25} \underbrace{f_1^0}_{0} = 0{,}14$

$V_2^1 = \lambda \left[V_3^0 + V_1^0 \right] + (1-2\lambda) V_2^0 + \frac{1}{25} f_2^0 = 0{,}28$

$V_1^2 = \lambda \left[V_2^1 + V_0^1 \right] + (1-2\lambda) V_1^1 + \frac{1}{25} \cdot \underbrace{f_1^1}_{\left(\frac{1}{3}\right)^2 + \frac{1}{4} + 1} = 0{,}16$

$f_1^2 = 1{,}69$

$\boxed{V_1^3 = 0{,}23}$

$V_4^0 = 0$

$V_2^0 = 1/4$

$V_3^0 = 0{,}56$

$f_3^0 = 0$

$V_3^1 = 0{,}37$

$V_0^0 = \frac{1}{4}$

$V_1^0 = 1/16$

$f_1^0 = 0$

$V_1^1 = 0{,}14$

$V_2^1 = 0{,}28$

$f_2^1 = 1{,}61$

$V_2^2 = 0{,}29$

$V_0^2 = 0{,}22$

$V_0^1 = 0{,}06$

$f_1^1 = 1{,}36$

30.09.2016

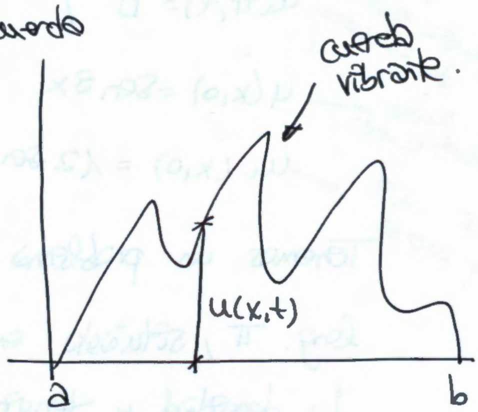
EC. DE LA ONDA. SOLUCIÓN EXACTA.

1.

$$\rho u_{tt} - \tau u_{xx} = f(x,t) \qquad a<x<b \, ; \, t>0$$

densidad (ρ)

tensión en la membrana de la cuerda



$f(x,t) \Rightarrow$ presiones exteriores aplicadas en los puntos del interior de la cuerda.

cond. de contorno

C.C. 1 $\quad u(a,t) = g_1(t)$

C.C. 2 $\quad u_x(b,t) = g_2(t)$

CI. 1 $\quad u(x,0) = h_0(x)$

CI. 2 $\quad u_t(x,0) = h_1(x)$

Cond. contorno

- Tipo DIRICHLET (desplazamiento)

$u(a,t) = g_1(t)$

$u(b,t) = g_2(t)$

- Tipo NEUMAN.

$u_x(a,t) = g_1(t)$

$u_x(b,t) = g_2(t)$

Cond. inicial (veloc. de desplazamiento de los puntos del interior de la cuerda).

$u(x,0) = h_0(x) \qquad a<x<b$

$u_t(x,0) = h_1(x) \qquad a<x<b$

(12)

densidad — tensión membrana.

$\delta u_{tt} - \partial u_{xx} = 0 \qquad 0 < x < \pi \ ; \ t > 0$

desplazamiento puntos extremos de la cuerda
$$\left.\begin{array}{l} u(0,t) = 0 \\ u(\pi,t) = 0 \end{array}\right\} t > 0$$

despl. puntos interiores cuerda. — $u(x,0) = \text{sen } 3x$

veloc. puntos interior cuerda. — $u_t(x,0) = 12 \cdot \text{sen } 6x$ $\quad \} \ 0 < x < \pi$

Tenemos un problema de ec. de la onda, de una cuerda vibrante de long. π, situada en el eje x entre 0 y π.
La densidad y tensión en la membrana de la cuerda vale 1.

Para $t>0$, el desplaz. de los extremos 0 y π vale 0. ; ó para todo $t>0$, los extremos están apoyados.
Para $t=0$, el desplaz. de los puntos del int. de la cuerda sigue la ley sen 3x, y para $t=0$, la veloc. de despl. de los ptos del interior de la cuerda siguen la ley 12 sen 6x.

M.S.V.

$u(x,t) = X(x) \cdot T(t)$

$X \cdot T'' - X'' \cdot T = 0$

$\dfrac{X''}{X} = \dfrac{T''}{T} = k$

$X'' - kX = 0$

$T'' - kT = 0$

$u(0,t) = 0 = X(0) \cdot T(t) \begin{cases} X(0) = 0 \\ T(t) = 0 \rightarrow S.T. \end{cases}$

$u(\pi,t) = 0 = X(\pi) \cdot T(t) \begin{cases} X(t) = 0 \\ T(t) = 0 \rightarrow ST. \end{cases}$

caso 1: $k=0$

$X''=0$

$\lambda^2=0;\ \lambda=\pm 0$

$X=C_1\cdot e^{0x}+C_2\cdot x\cdot e^{0x}=C_1+C_2x$

$X(0)=0=C_1+C_2x \rightarrow C_1=0$

$X(\pi)=0=C_2\pi \rightarrow C_2=0$

$X=0$

$u(x,t)=0$

caso 2: $k>0$

$X''-kX=0$

$\lambda^2-k=0;\ \lambda=\pm\sqrt{k}$

$X=C_1\cdot e^{\sqrt{k}\cdot x}+C_2\cdot e^{-\sqrt{k}\cdot x}$

$X(0)=0=C_1\cdot e^0+C_2\cdot e^0$

$X(\pi)=0=C_1\cdot e^{\sqrt{k}\pi}+e^{-\sqrt{k}\cdot\pi}$

$C_1=C_2=0 \rightarrow X=0$

$u(x,t)=0$

caso 3: $k<0$

$X''-kX=0$

$\lambda^2-k=0;\ \lambda=\pm\sqrt{k}=\pm\sqrt{-k}\cdot i$

$X=C_1\cancel{e^{0x}}\cdot\cos\sqrt{-k}\cdot x+C_2\cdot e^{0x}\cdot\operatorname{sen}\sqrt{-k}\cdot x$

$X(0)=0=C_1\cdot\cos 0+C_2\cdot\operatorname{sen}0 \rightarrow C_1=0$ } ST

$X(\pi)=0=C_2\cdot\operatorname{sen}\sqrt{-k}\cdot\pi$ < $C_2=0$; $\operatorname{sen}\sqrt{-k}\cdot\pi=0$

$\sqrt{-k}\cdot\cancel{\pi}=n\cancel{\pi};\ n=1,2\dots$

$$X = C_2 \cdot \operatorname{sen} nx$$

$$X = \sum_{n=1}^{\infty} C_2 \cdot \operatorname{sen} nx$$

$$T'' - kT = 0$$

$$\lambda^2 - k = 0; \ \lambda = \pm\sqrt{k} = \pm\sqrt{-k} \cdot i$$

$$T = C_3 \cdot e^{0t} \cdot \cos\sqrt{-k} \cdot t + C_4 \cdot e^{0t} \cdot \operatorname{sen}\sqrt{-k} \cdot t$$

$$T = C_3 \cdot \cos nt + C_4 \cdot \operatorname{sen} nt$$

$$T = \sum_{n=1}^{\infty} C_3 \cdot \cos nt + C_4 \cdot \operatorname{sen} nt$$

$$u(x,t) = \sum_{n=1}^{\infty} \left[\underset{C_2 \cdot C_3}{\underset{\shortparallel}{D_n}} \cdot \operatorname{sen} nx \cdot \cos nt + \underset{C_2 \cdot C_4}{\underset{\shortparallel}{E_n}} \cdot \operatorname{sen} nx \cdot \operatorname{sen} nt \right]$$

$$u(x,0) = \operatorname{sen} 3x = \sum_{n=1}^{\infty} \left[D_n \cdot \operatorname{sen} nx \cdot \cos 0 + E_n \cdot \operatorname{sen} nx \cdot \underset{0}{\underset{\shortparallel}{\operatorname{sen} 0}} \right]$$

$$\operatorname{sen} 3x = \sum_{n=1}^{\infty} D_n \cdot \operatorname{sen} nx$$

$$\operatorname{sen} 3x = D_1 \cdot \operatorname{sen} x + D_2 \cdot \operatorname{sen} 2x + D_3 \cdot \operatorname{sen} 3x + \ldots$$

$$\boxed{\begin{array}{l} D_3 = 1 \\ D_n \ (n \neq 3) = 0 \end{array}}$$ solución

derivando respecto a "t"

$$u_t(x,0) = 12 \cdot \operatorname{sen} 6x \ ; \ u_t(x,t) = \sum_{n=1}^{\infty} [D_n \cdot \operatorname{sen} nx \cdot n \cdot \operatorname{sen} nt +$$

$$+ E_n \cdot \operatorname{sen} nx \cdot n \cdot \cos nt]$$

$$12 \operatorname{sen} 6x = \sum_{n=1}^{\infty} \left[-n \cdot D_n \cdot \operatorname{sen} nx \cdot \underset{0}{\underset{\shortparallel}{\operatorname{sen} 0}} + n \cdot E_n \cdot \operatorname{sen} nx \cdot \underset{1}{\underset{\shortparallel}{\cos 0}} \right]$$

$$12 \operatorname{sen} 6x = \sum_{n=1}^{\infty} n \cdot E_n \cdot \operatorname{sen} nx = 1 \cdot E_1 \cdot \operatorname{sen} x + 2 \cdot E_2 \cdot \operatorname{sen} 2x + \ldots +$$

$$+ 6 \cdot E_6 \cdot \operatorname{sen} 6x \longrightarrow 12 = 6E_6 \longrightarrow \boxed{\begin{array}{l} E_6 = 2 \\ E_n \ (n \neq 6) = 0 \end{array}}$$ solución

30.09.2016

3.

$$u(x,t) = \underset{n=1}{0} + \underset{n=2}{0} + \underset{n=3}{1\cdot \operatorname{sen} 3x \cos 3t} + 0 + \underset{n=4}{0} + \underset{n=5}{0} +$$

$$+ \underset{n=6}{2\cdot \operatorname{sen} 6x \cdot \operatorname{sen} 6t} + \underset{n=7}{0}$$

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03.10.2016

$3u_{tt} - 0{,}75\, u_{xx} = 0 \qquad 0<x<2\,;\ t>0$

$\left.\begin{array}{l} u(0,t)=0 \\ u(2,t)=0 \end{array}\right\} t>0$

$u(0,t) = 0 = X(0)\cdot T(t) \;\; \begin{cases} X(0)=0 \\ T(t)=0 \text{ S.T.} \end{cases}$

$\left.\begin{array}{l} u(x,0) = \pi\cdot \operatorname{sen} \frac{\pi x}{2} \\ u_t(x,0) = \cos \frac{\pi x}{2} \end{array}\right\} 0<x<2$

$u(2,t) = 0 = X(2)\cdot T(t) \;\; \begin{cases} X(2)=0 \\ T(t)=0 \text{ S.T.} \end{cases}$

$u(x,t) = X(x)\cdot T(t)$

$3XT'' - 0{,}75\cdot X''\cdot T = 0$

$\dfrac{X''}{X} = \dfrac{T''}{\frac{0{,}75}{3}T} = k$

$\left.\begin{array}{l} X'' - kX = 0 \\ T'' - 0{,}25\, kT = 0 \end{array}\right\}$

$\underline{k=0}$

$X''=0$

$\lambda^2=0\,;\ \lambda=\pm 0$

$X = C_1\cdot e^{0x} + C_2\cdot x\cdot e^{0x}$

$X(0) = 0 = C_1 + C_2\cdot 0 \rightarrow C_1 = 0$

$X(2) = 0 = C_2\cdot 2 \longrightarrow C_2 = 0$

$X = 0$

$u(x,t) = 0$

$\underline{k>0}$

$$X'' - kX = 0$$

$$\lambda^2 - k = 0 \; ; \quad \lambda = \pm\sqrt{k}$$

$$X = C_1 \cdot e^{\sqrt{k}\cdot x} + C_2 \cdot e^{-\sqrt{k}\cdot x}$$

$$\left.\begin{aligned} X(0) &= 0 = C_1 \cdot e^0 + C_2 \cdot e^0 \\ X(2) &= 0 = C_1 \cdot e^{\sqrt{k}\cdot 2} + C_2 \cdot e^{-\sqrt{k}\cdot 2} \end{aligned}\right\} \quad \begin{aligned} &C_1 = C_2 = 0 \\ &X = 0, \quad u(x,t) = 0 \end{aligned}$$

$\underline{k<0}$

$$X'' - kX = 0$$

$$\lambda^2 - k = 0; \quad \lambda = \pm\sqrt{k} = \pm\sqrt{-k}\cdot i$$

$$X = C_1 \cdot e^{0x} \cdot \cos\sqrt{-k}\cdot x + C_2 \cdot e^{0x} \cdot \operatorname{sen}\sqrt{-k}\cdot x$$

$$\left.\begin{aligned} X(0) &= 0 = C_1 \cdot \underbrace{\cos 0}_{1} + C_2 \cdot \underbrace{\operatorname{sen} 0}_{0} \longrightarrow C_1 = 0 \\ X(2) &= 0 = C_2 \cdot \operatorname{sen}\sqrt{-k}\cdot 2 \longrightarrow C_2 = 0 \end{aligned}\right\} \text{S.T.}$$

$$\operatorname{sen}\sqrt{-k}\cdot 2 = 0$$

$$\sqrt{-k}\cdot 2 = n\pi \; ; \quad n = 1, 2 \ldots$$

$$\sqrt{-k} = \frac{n\pi}{2}$$

$$X = C_2 \cdot \operatorname{sen}\frac{n\pi}{2} x$$

$$X = \sum_{n=1}^{\infty} C_2 \cdot \operatorname{sen}\frac{n\pi}{2}\cdot x$$

$$T'' - 0{,}25\, kT = 0$$

$$\lambda^2 - 0{,}25k = 0$$

$$\lambda = \pm\sqrt{0{,}25k} = 0 \pm 0{,}5\sqrt{-k}\cdot i$$

$$T = C_3 \cdot e^{0t} \cdot \cos 0{,}5\sqrt{-k}\cdot t + C_4 \cdot e^{0t} \cdot \operatorname{sen} 0{,}5\sqrt{-k}\cdot t$$

$$T = C_3 \cdot \cos\frac{n\pi t}{4} + C_4 \cdot \operatorname{sen}\frac{n\pi t}{4}$$

$$T = \sum_{n=1}^{\infty} \left[C_3 \cdot \cos \frac{n\pi t}{4} + C_4 \cdot \operatorname{sen} \frac{n\pi t}{4} \right]$$

$$u(x,t) = \sum_{n=1}^{\infty} \left[D_n \cdot \operatorname{sen} \frac{n\pi x}{2} \cdot \cos \frac{n\pi t}{4} + E_n \cdot \operatorname{sen} \frac{n\pi x}{2} \cdot \operatorname{sen} \frac{n\pi t}{4} \right]$$

$$u(x,0) = \pi \cdot \operatorname{sen} \frac{\pi x}{2} = \sum_{n=1}^{\infty} \left[D_n \cdot \operatorname{sen} \frac{n\pi x}{2} \cdot \underset{1}{\underset{\shortparallel}{\cos 0}} + E_n \cdot \operatorname{sen} \frac{n\pi x}{2} \cdot \underset{0}{\underset{\shortparallel}{\operatorname{sen} 0}} \right]$$

$$\pi \cdot \underbrace{\operatorname{sen} \frac{\pi x}{2}} = \sum_{n=1}^{\infty} D_n \cdot \operatorname{sen} \frac{n\pi x}{2} = D_1 \cdot \underbrace{\operatorname{sen} \frac{\pi x}{2}} + D_2 \cdot \operatorname{sen} \frac{2\pi x}{2} + D_3 \cdot \operatorname{sen} \frac{3\pi x}{2} + \dots$$

$$D_1 = \pi$$
$$D_n \ (n \neq 1) = 0$$

$$u_t(x,t) = \sum_{n=1}^{\infty} \left[-\frac{n\pi}{4} \cdot D_n \cdot \operatorname{sen} \frac{n\pi x}{2} \cdot \operatorname{sen} \frac{n\pi t}{4} + \frac{n\pi}{4} \cdot E_n \cdot \operatorname{sen} \frac{n\pi x}{2} \cdot \cos \frac{n\pi t}{4} \right]$$

$$u_t(x,0) = \cos \frac{\pi x}{2} = \sum_{n=1}^{\infty} \left[-\frac{n\pi}{4} \cdot D_n \cdot \operatorname{sen} \frac{n\pi x}{2} \cdot \underset{0}{\underset{\shortparallel}{\operatorname{sen} 0}} + \frac{n\pi}{4} \cdot E_n \cdot \operatorname{sen} \frac{n\pi x}{2} \cdot \underset{1}{\underset{\shortparallel}{\cos 0}} \right]$$

$$\cos \frac{\pi x}{2} = \sum_{n=1}^{\infty} \frac{n\pi}{4} \cdot E_n \cdot \operatorname{sen} \frac{n\pi x}{2}$$

$$\boxed{\frac{n\pi}{4} \cdot E_n} = \frac{1}{2/2} \int_0^2 \cos \frac{\pi x}{2} \cdot \operatorname{sen} \frac{n\pi x}{2} \cdot dx = \frac{1}{2} \int_0^2 \left[\operatorname{sen} (1-n) \cdot \frac{\pi x}{2} + \right.$$

$$\left. + \operatorname{sen} (1+n) \cdot \frac{\pi x}{2} \right] \cdot dx =$$

$$= \frac{1}{2} \cdot \frac{1}{(1+n) \cdot \frac{\pi}{2}} \int_0^2 (1-n) \frac{\pi}{2} \cdot \operatorname{sen} (1-n) \cdot \frac{\pi x}{2} \cdot dx + \frac{1}{2} \cdot \frac{1}{(1+n) \frac{\pi}{2}} \int_0^2 (1+n) \cdot \operatorname{sen} (1+n) \cdot \frac{\pi}{2} \cdot x \cdot dx =$$

$D_1 = \pi$
$D_n (n \neq 1) = 0$

$$= \frac{-1}{(1-n) \cdot \pi} \cos (1-n) \cdot \frac{\pi x}{2} \Big|_0^2 - \frac{1}{(1+n) \pi} \cdot \cos (1+n) \cdot \frac{\pi x}{2} \Big|_0^2 =$$

$$= \boxed{\frac{-1}{(1-n)\pi} \left[\cos (1-n) \cdot \frac{\pi}{2} \cdot 2 - 0 \right] - \frac{1}{(1+n)\pi} \left[\cos (1+n) \cdot \frac{\pi}{2} \cdot 2 - 0 \right]} \quad (*)$$

$$E_n = \frac{4}{n\pi} \cdot (*)$$

$$u(x,t) = \pi \cdot \operatorname{sen} \frac{\pi x}{2} \cdot \cos \frac{\pi t}{4} + \sum_{n=1}^{\infty} \frac{4}{n\pi} (*) \cdot \operatorname{sen} \frac{n\pi x}{2} \cdot \operatorname{sen} \frac{n\pi t}{4}$$

EC. DE LA ONDA. SOLUCIÓN APROXIMADA.

Método explícito:

$$V_e^{n+1} = (2-2\lambda) V_e^n + \lambda \left(V_{e+1}^n + V_{e-1}^n \right) - V_e^{n-1} + \frac{k^2}{\rho} \cdot f_e^n$$

$$\lambda = \frac{\tau \cdot k^2}{\rho h^2}$$

Método implícito:

$$(1+2\lambda) V_e^{n+1} - \lambda \left[V_{e+1}^{n+1} + V_{e-1}^{n+1} \right] = 2 V_e^n - V_e^{n-1} + \frac{k^2}{\rho} \cdot f_e^n$$

$n = 1$ $\quad$ $e = 1$

$e = 2$

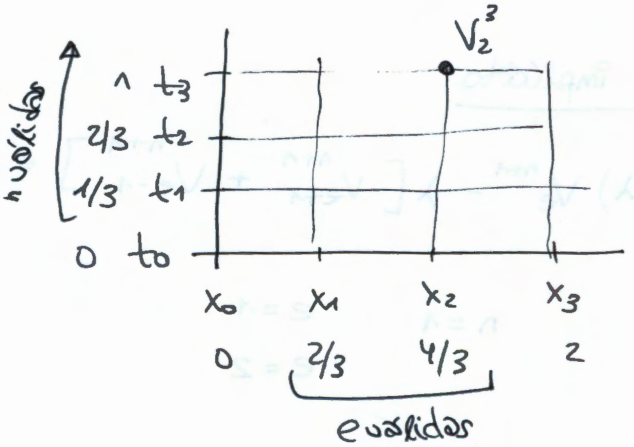
$$15u_{tt} - u_{xx} = 2t^2 + x \;;\quad 0<x<2 \;,\; t>0$$

$$\left.\begin{array}{l} u(0,t) = t+1 \\ u_x(2,t) = t^2 \end{array}\right\} t>0 \qquad \frac{2}{2/3} = 3 \;;\; e = 0,1,2,3$$

$$\left.\begin{array}{l} u(x,0) = x^2-3 \\ u_t(x,0) = x-2 \end{array}\right\} 0<x<2 \qquad \frac{1}{1/3} = 3 \;;\; n = 0,1,2,3$$

MDF implícito ¿$u\left(\frac{4}{3},1\right)$?

$$h = \frac{2}{3} \;;\; k = \frac{1}{3}$$



$$u(0,t) \simeq u(x_0,t_n) = V_0^n = t_n + 1 \;;\; n = 1,2\ldots$$

$$u_x(2,t) \simeq u_x(x_3,t_n) = \frac{V_2^n - V_3^n}{-2/3} = t_n^2 \qquad \lambda = \frac{1}{60}$$

$$u(x,0) \simeq u(x_e,t_0) = V_e^0 = X_e^2 - 3 \;,\; e = 1,2$$

$$\frac{k^2}{\rho} = \frac{1}{135}$$

$$u_t(x,0) \simeq u_t(x_e,t_0) = \frac{V_e^1 - V_e^0}{1/3} = X_e - 2$$

$$f(x,t) \simeq f(x_e,t_n) = f_e^n = 2t_n^2 + X_e \quad \left\{\begin{array}{l} e = 1,2 \\ n = 1,2\ldots \end{array}\right.$$

$$(1+2\lambda)V_e^{n+1} - \lambda\left[V_{e+1}^{n+1} + V_{e-1}^{n+1}\right] = 2V_e^n - V_e^{n-1} + \frac{k^2}{\rho}\cdot f_e^n$$

$$\left.\begin{array}{l} V_0^n = t_n + 1 \\ V_2^n - V_3^n = -\frac{2}{3}t_n^2 \end{array}\right\} n = 1,2\ldots$$

$$\left.\begin{array}{l} V_e^0 = X_e^2 - 3 \\ V_e^1 = X_e^2 + \frac{1}{3}\cdot X_e - \frac{11}{3} \end{array}\right\} e = 1,2$$

$$f_e^n = 2t_n^2 + X_e \quad \left\{\begin{array}{l} e = 1,2 \\ n = 1,2\ldots \end{array}\right.$$

10.10.2016

$$n=1 \quad e=1 \quad (1+2\lambda) V_1^2 - \lambda [V_2^2 + \underset{\frac{2}{3}+1}{V_0^2}] = 2\underset{-3}{V_1^1} - \underset{-2{,}55}{V_1^0} + \frac{1}{135}\cdot \underset{0{,}89}{f_1^1} \quad (1)$$

③

$$V_1^1 = \left(\frac{2}{3}\right)^2 + \frac{1}{3}\cdot\frac{2}{3} - \frac{11}{3} = -3$$

$$V_1^0 = \left(\frac{2}{3}\right)^2 - 3 = -2{,}55$$

$$f_1^1 = 2\left(\frac{1}{3}\right)^2 + \frac{2}{3} = 0{,}89 \quad ; \quad f_2^1 = 1{,}55 \quad ; \quad f_1^2 = 1{,}56 \quad ; \quad f_2^2 = 2{,}22$$

$$e=2 \quad (1+2\lambda)\cdot V_2^2 - \lambda [\underset{V_2^2 + \frac{2}{3}\cdot\left(\frac{2}{3}\right)^2}{V_3^2} + V_1^2] = 2\underset{-1{,}44}{V_2^1} - \underset{-1{,}22}{V_2^0} + \frac{1}{135}\cdot \underset{1{,}55}{f_2^1} \quad (2)$$

Resolviendo (1) y (2)

$$\begin{cases} V_1^2 = -3{,}34 \\ V_2^2 = -1{,}67 \end{cases}$$

$$n=2 \quad e=1 \quad (1+2\lambda)\boxed{V_1^3} - \lambda [\boxed{V_2^3} + \underset{1+1}{V_0^3}] = 2\underset{-3{,}34}{V_1^2} - \underset{-3}{V_1^1} + \frac{1}{135}\underset{1{,}56}{f_1^2}$$

$$e=2 \quad (1+2\lambda) V_2^3 - \lambda [\underset{V_2^3 + \frac{2}{3}\cdot 1^2}{V_3^3} + V_1^3] = 2\underset{-1{,}67}{V_2^2} - \underset{-1{,}44}{V_2^1} + \frac{1}{135}\cdot\underset{2{,}22}{f_2^2}$$

$$V_1^3 = -3{,}56$$

$$\boxed{V_2^3 = -1{,}89}$$

EXAMEN.

$$3u_{tt} - 2u_{xx} = t^2 + x \qquad -\pi < x < 2\pi \; ; \; t > 0$$

$$\left.\begin{aligned} u_x(-\pi,t) &= t+1 \\ u(2\pi,t) &= 2t \end{aligned}\right\} t > 0$$

$$\frac{3\pi}{\pi} = 3 \; ; \; e = 0,1,2,3$$

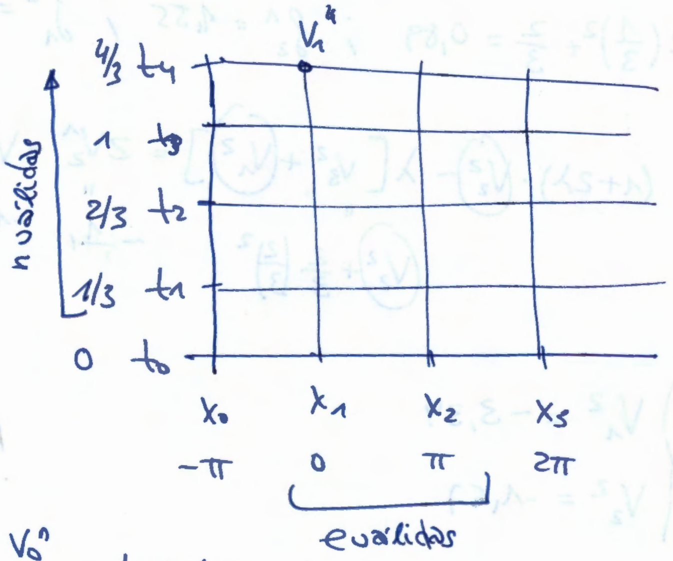
$$\left.\begin{aligned} u(x,0) &= \cos x \\ u_t(x,0) &= \operatorname{sen} x \end{aligned}\right\} -\pi < x < 2\pi$$

$$\frac{4/3}{1/3} = 4 \; ; \; n = 0,1,2,3,4$$

MDF expl. ¿$u(0, \frac{4}{3})$?

$h = \pi$; $k = \frac{1}{3}$

Dato: $f_e^0 = f_e^1$



$$u_x(-\pi,t) \simeq u_x(x_0,t_n) = \frac{V_1^n - V_0^n}{\pi} = t_n + 1 \; ; \; n = 1,2\ldots$$

$$u(2\pi,t) \simeq u(x_3,t_n) = V_3^n = 2t_n$$

$e=1$ $V_1^0 = \cos x_1$

$e=2$ $V_2^0 = \cos x_2$

$V_e^0 = \cos x_e$

$$u(x,0) \simeq u(x_e,t_0) = V_e^0 = \cos x_e \; ; \; e = 1,2$$

$$u_t(x,0) \simeq u_t(x_e,t_0) = \frac{V_e^1 - V_e^0}{1/3} = \operatorname{sen} x_e$$

$$\lambda = 0{,}0075$$

$$\frac{k^2}{e} = \frac{1}{27} = 0{,}037$$

$$\left.\begin{aligned} V_1^n - V_0^n &= \pi(t_n + 1) \\ V_3^n &= 2t_n \end{aligned}\right\} n = 1,2\ldots$$

$$\left.\begin{aligned} V_e^0 &= \cos x_e \\ V_e^1 &= \frac{1}{3}\cdot \operatorname{sen} x_e + \cos x_e \end{aligned}\right\} e = 1,2$$

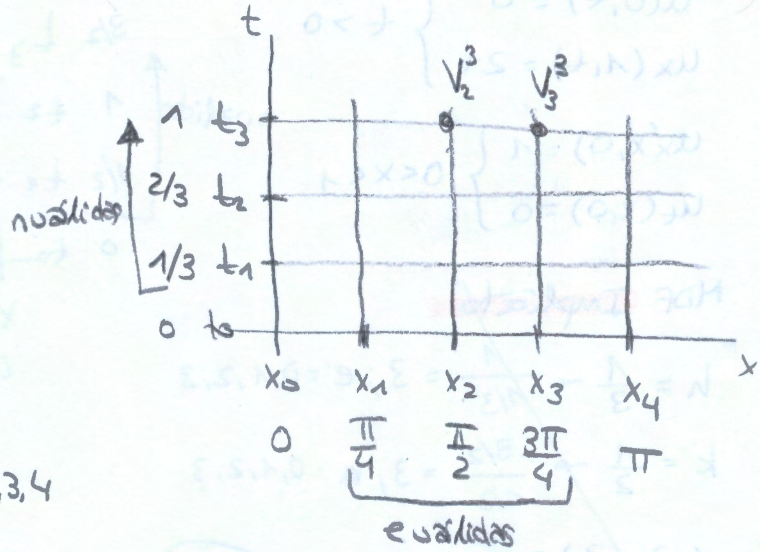
$$f_e^n = t_n^2 + x_e \quad \left\} \begin{aligned} e &= 1,2 \\ n &= 1,2\ldots \end{aligned}\right.$$

(18) **ONDA**

$$u_{tt} - \frac{1}{4}u_{xx} = 0 \qquad 0<x<\pi \ ; \ t>0$$

$$\left.\begin{aligned} u(0,t) &= 0 \\ u(\pi,t) &= 0 \end{aligned}\right\} t>0$$

$$\left.\begin{aligned} u(x,0) &= x \\ u_t(x,0) &= \operatorname{sen} x \end{aligned}\right\} 0<x<\pi$$



MDF <u>explícito</u> ¿$u\left(\frac{\pi}{2},1\right)$? ¿$u\left(\frac{3\pi}{4},1\right)$?

$h = \frac{\pi}{4} \rightarrow \frac{\pi}{\pi/4} = 4 \ ; \ e = 0,1,2,3,4$

$k = \frac{1}{3} \rightarrow \frac{1}{1/3} = 3 \ ; \ n = 0,1,2,3$

$\lambda = 0{,}045$

$\frac{k^2}{\rho} = 0{,}11$

$$\left.\begin{aligned} u(0,t) \simeq u(x_0,t_n) &= V_0^n = 0 \\ u(\pi,t) \simeq u(x_4,t_n) &= V_4^n = 0 \end{aligned}\right\} n = 1,2,\dots$$

$$\left.\begin{aligned} u(x,0) \simeq u(x_e,t_0) &= V_e^0 = x_e \\ u_t(x,0) \simeq u(x_e,t_0) &= \frac{V_e^1 - V_e^0}{1/3} = \operatorname{sen} x_e \end{aligned}\right\} \begin{aligned} e &= 1,2,3 \\ n &= 1,2\dots \end{aligned} \rightarrow \boxed{V_e^1 = \frac{1}{3}\operatorname{sen} x_e + x_e}$$

$$f(x,t) \simeq f(x_e,t_n) = f_e^n = 0 \quad \begin{cases} e = 1,2,3 \\ n = 1,2\dots \end{cases}$$

$$V_e^{n+1} = (2-2\lambda)V_e^n + \lambda\left[V_{e+1}^n + V_{e-1}^n\right] - V_e^{n-1} + \cancel{\frac{k^2}{\rho} f_e^n}^{\,0}$$

$$\boxed{V_2^3} = (2-2\lambda)\underset{2{,}22}{V_2^2} + \lambda\left[\underset{2{,}68}{V_3^2} + \underset{1{,}25}{V_1^2}\right] - \underset{1{,}9}{V_2^1} + \cancel{0{,}11 f_2^2}^{\,0} = \boxed{2{,}51}$$

$$V_2^2 = (2-2\lambda)\underset{1{,}9}{V_2^1} + \lambda\left[\underset{2{,}59}{V_3^1} + \underset{1{,}02}{V_1^1}\right] - \underset{1{,}57}{V_2^0} + \cancel{0{,}11 f_2^1}_{\,0} = 2{,}22$$

$$V_3^2 = (2-2\lambda)\underset{2{,}59}{V_3^1} + \lambda\left[\underset{0}{V_4^1} + \underset{1{,}9}{V_2^1}\right] - \underset{2{,}35}{V_3^0} = 2{,}68$$

$$V_1^2 = (2-2\lambda)\underset{1{,}02}{V_1^1} + \lambda\left[\underset{1{,}9}{V_2^1} + \underset{0}{V_0^1}\right] - \underset{0{,}78}{V_1^0} = 1{,}25$$

$$\boxed{V_3^3} = (2-2\lambda)\underset{2{,}68}{V_3^2} + \lambda\left[\underset{0}{V_4^2} + \underset{2{,}22}{V_2^2}\right] - \underset{2{,}59}{V_3^1} = \boxed{2{,}63}$$

$V_2^1 = 1{,}9$

$V_3^1 = 2{,}59$

$V_1^1 = 1{,}02$

$V_2^0 = 1{,}57$

$V_2^2 = 2{,}22$

$V_4^1 = 0$

$V_3^0 = 2{,}35$

$V_3^2 = 2{,}68$

$V_0^1 = 0$

$V_1^0 = 0{,}78$

$V_1^2 = 1{,}25$

$V_2^3 = 2{,}51$

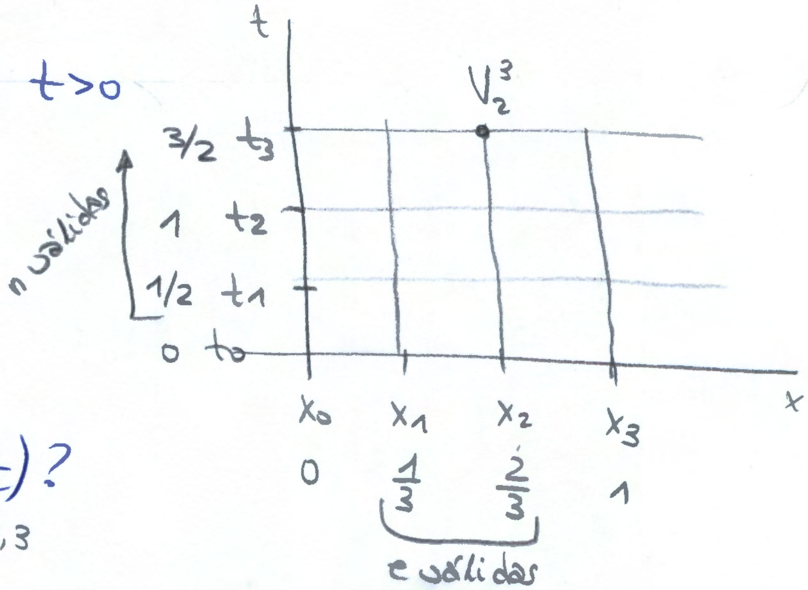
$V_4^2 = 0$

21 ONDA.

$$2u_{tt} - u_{xx} = t^2 \qquad 0<x<1;\ t>0$$

$$\left.\begin{aligned} u(0,t) &= 0 \\ u_x(1,t) &= 2t \end{aligned}\right\} t>0$$

$$\left.\begin{aligned} u(x,0) &= 1 \\ u_t(x,0) &= 0 \end{aligned}\right\} 0<x<1$$



MDF implícito ¿$u\left(\frac{2}{3}, \frac{3}{2}\right)$?

$h = \frac{1}{3} \to \frac{1}{1/3} = 3;\ e = 0,1,2,3$

$k = \frac{1}{2} \to \frac{3/2}{1/2} = 3;\ n = 0,1,2,3$

$$\left.\begin{aligned} u(0,t) &\simeq u(x_0,t_n) = V_0^n = 0 \\ u_x(1,t) &\simeq u_x(x_3,t_n) = \frac{V_2^n - V_3^n}{-1/3} = 2t_n \end{aligned}\right\} n=1,2\ldots \qquad \left.\begin{aligned} V_0^n &= 0 \\ V_2^n - V_3^n &= -\frac{2}{3}t_n \end{aligned}\right\} n=1,2\ldots$$

$$\left.\begin{aligned} u(x,0) &\simeq u(x_e,t_0) = V_e^0 = 1 \\ u_t(x,0) &\simeq u_t(x_e,t_0) = \frac{V_e^1 - V_e^0}{1/2} = 0 \end{aligned}\right\} e=1,2 \qquad \left.\begin{aligned} V_e^0 &= 1 \\ V_e^1 &= V_e^0 = 1 \end{aligned}\right\} e=1,2$$

$$f(x,t) \simeq f(x_e,t_n) = f_e^n = t_n^2 \left\{\begin{aligned} e &= 1,2 \\ n &= 1,2\ldots \end{aligned}\right. \qquad f_e^n = t_n^2 \left\{\begin{aligned} e &= 1,2 \\ n &= 1,2\ldots \end{aligned}\right.$$

$\lambda = 1{,}125$

$f_1^1 = 0{,}25 \quad V_0^2 = 0$

$f_2^1 = 0{,}25 \quad V_1^1 = 1$

$f_1^2 = 1 \quad V_1^0 = 1$

$f_2^2 = 1 \quad V_2^1 = 1$

$V_2^0 = 1$

$V_0^3 = 0$

$\boxed{n=1 \quad e=1}$ $(1+2\lambda)V_e^{n+1} - \lambda(V_{e+1}^{n+1} + V_{e-1}^{n+1}) = \frac{k^2}{\rho} \cdot f_e^n + 2V_e^n - V_e^{n-1}$ $\quad \frac{k^2}{\rho} \to 0{,}125$

$$(1+2\lambda)V_1^2 - \lambda(V_2^2 + \underset{0}{V_0^2}) = 0{,}125 \cdot \underset{0{,}25}{f_1^1} + 2 \cdot \underset{1}{V_1^1} - \underset{1}{V_1^0}$$

$$3{,}25V_1^2 - 1{,}125V_2^2 = 1{,}03$$

$\boxed{e=2}$ $(1+2\lambda)V_2^2 - \lambda(V_3^2 + V_1^2) = 0{,}125 f_2^1 + 2V_2^1 - V_2^0$

$$V_3^2 = V_2^2 + \frac{2}{3}\cdot 1, \quad f_2^1 = 0{,}25, \quad V_2^1 = 1, \quad V_2^0 = 1$$

$$3{,}25V_2^2 - 1{,}125V_2^2 - 1{,}125V_1^2 = 1{,}78$$

$$\left.\begin{aligned} V_1^2 &= 0{,}74 \\ V_2^2 &= 1{,}23 \end{aligned}\right.$$

$\boxed{n=2 \quad e=1}$ $(1+2\lambda)V_1^3 - \lambda(V_2^3 + \underset{0}{V_0^3}) = 0{,}125 \underset{1}{f_1^2} + 2\underset{0{,}74}{V_1^2} - \underset{1}{V_1^1}$

$$3{,}25V_1^3 - 1{,}125V_2^3 = 0{,}605$$

$\boxed{e=2}$ $(1+2\lambda)V_2^3 - \lambda(V_3^3 + V_1^3) = 0{,}125 \underset{1}{f_2^2} + 2\underset{1{,}23}{V_2^2} - \underset{1}{V_2^1}$

$$V_3^3 = V_2^3 + \frac{2}{3}\cdot\frac{3}{2}$$

$$3{,}25V_2^3 - 1{,}125V_2^3 - 1{,}125V_1^3 = 2{,}71$$

$$V_1^3 = 0{,}77$$

$$\boxed{V_2^3 = 1{,}68}$$